



# Polynomial representation of functions on the integers modulo $n$

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**Abstract.** We define a polynomial index  $PI$  for any finite commutative ring with unity element. This index provides a measure of the distance the ring is from being a finite field (whose  $PI$  is 1). After proving a multiplicativity property for the ring  $Z_n$ , we focus on the case of  $Z_{p^m}$  ( $p$  a prime). We determine the index for this ring using the concepts of annihilator polynomials and stopping point degrees. Finally, we give a specific formula for  $PI(Z_{p^m})$  in terms of  $p$  and  $m$  only, provided that  $m \leq p$ .

## References

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