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Abstract. A 2-factor of a graph G is a 2-regular spanning subgraph of G. We present a survey summarising results on the structure of 2-factors in regular graphs, as achieved by various researchers in recent years.

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