



# Cozonal Labelings of Plane Graphs

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**Abstract.** A cozonal labeling of a plane graph  $G$  is a labeling  $\ell^* : F(G) \rightarrow \{1, 2\} \subset \mathbb{Z}_3$  such that for  $X_v$  (the set of regions with  $v$  on their boundary),  $\sum_{R \in X_v} \ell^*(R) = 0$ . This particular labeling is a dual of zonal labelings, which have an important connection to the Four-Color Theorem. In this paper, we determine relationships between zonal and cozonal labelings, establish some results on cozonal labelings, and characterize cozonal graphs of maximum degree at most 3.

## References

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