## Computational lower bounds for weakened Ramsey numbers from strongly regular *t*-colorings of complete graphs

MARK BUDDEN, JOSH HILLER, AND LASZLO GOCH

Abstract. We introduce and study strongly regular *t*-colorings of complete graphs and prove some properties which extend well-known properties of strongly regular graphs. As an example, we consider a coloring that corresponds with a partition of the edges of a complete graph into congruence classes modulo the set of  $t^{th}$  powers modulo a given prime number. We then show how a modified version of an algorithm for cubic residue graphs due to Su, Li, Luo, and Li (2002) can be used computationally to find new lower bounds for 26 weakened Ramsey numbers.

## References

- [1] J. M. Aija'am, Can Genetic Algorithms with the Symmetric Heuristic Find the Ramsey Number R(5,5), The 7<sup>th</sup> International Conference on Informatics and Systems (INFOS), Cairo, Egypt (2010), 1–8.
- [2] G. Beam and M. Budden, Weakened Gallai-Ramsey Numbers, Surv. Math. Appl, 13 (2018), 131–145.
- [3] M. Budden, M. Stender, and Y. Zhang, Weakened Ramsey Numbers and Their Hypergraph Analogues, *Integers* 17 (2017), #A23.
- [4] M. Budden and T. Wimbish, Subgraphs of Gallai-Colored Complete Graphs Spanned by Edges Using at Most Two Colors, Australas. J. Combin., 84 (2022), 375–387.
- [5] K. Chung, M. Chung, and C. Liu, A Generalization of Ramsey Theory for Graphs - with Stars and Complete Graphs as Forbidden Subgraphs, *Congr. Numer.*, **19** (1977), 155–161.
- [6] K. Chung and C. Liu, A Generalization of Ramsey Theory for Graphs, Discrete Math., 21 (1978), 117–127.
- [7] C Godsil and G. Royle, Algebraic Graph Theory, Springer-Verlag, New York, 2001.

Received: 1 November 2024 Accepted: 8 May 2025

- [8] R. Greenwood and A. Gleason, Combinatorial Relations and Chromatic Graphs, Canadian J. Math., 7 (1955), 1–7.
- [9] H. Harborth and M. Möller, Weakened Ramsey Numbers, Discrete Appl. Math., 95 (1999), 279–284.
- [10] M. Jacobson, On a Generalization of Ramsey Theory, Discrete Math. 38 (1982), 191–195.
- [11] W. Mao, F. Gao, Y. Dong, and W. Li, A Novel Paradigm for Calculating Ramsey Number Via Artificial Bee Colony Algorithm, 35<sup>th</sup> Chinese Control Conference (CCC), Chengdu, China (2016), 9189–9195.
- [12] S. Radziszowski, Small Ramsey Numbers Revision #17, Electron. J. Combin. DS1.17 (2024), 133 pages.
- [13] W. Su, Q. Li, H. Luo, and G. Li, Lower Bounds of Ramsey Numbers Based on Cubic Residues, *Discrete Math.*, 250 (2002), 197–209.
- [14] X. Xu and S. Radzisowski, Bounds on Shannon Capacity and Ramsey Numbers From Product of Graphs, *IEEE Trans. Inform. Theory*, 59(8) (2013), 4767–4770.

MARK BUDDEN DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE WESTERN CAROLINA UNIVERSITY CULLOWHEE, NC 28723 USA mrbudden@email.wcu.edu

Josh Hiller Department of Mathematics and Computer Science Adelphi Univeristy Garden City, NY, USA johiller@adelphi.edu

LASZLO GOCH DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE WESTERN CAROLINA UNIVERSITY ADELPHI UNIVERISTY laszlogoch@mail.adelphi.edu