BULLETIN OF THE ICA
Article in press.

The number of paths in uniform cactus chains

Pennapa Chodok, Pitsinee Meeboon,
Pawaton Kaemawichanurat, and Natawat Klamsakul

Abstract. A graph G is called g-gonal cactus if G has two end blocks and every block of G is C_g, a cycle of g vertices. This paper aims to establish the generating function and the recurrence relation to count the total number of paths of all g-gonal cacti. Surprisingly, all g-gonal cacti (even the random structure) have the same total number of paths. By analyzing the Laurent series of the generating function, we derived the asymptotic formula for the total number of such paths as well. Furthermore, we constructed the formulae to calculate the number of paths of a given length of regular g-gonal cacti. The formulae are implemented in Python and provided in this paper.

References

[1] T. Došlić and M. S. Litz, Matchings and independent sets in polyphenylene chains. *MATCH Commun. Math. Comput. Chem.*, **67** (2011), 314–329.

[2] T. Došlić and I. Zubac, Counting maximal matchings in linear polymers. *Ars Math. Contemp.*, **11**(2) (2015), 256–265.

[3] T. Došlić and F. Måløy, Chain hexagonal cacti: Matching and independent sets, *Discrete Math.* , **310** (2010), 1676–1690.

[4] N. Movarraei and S. A. Boxwala, On the number of paths of length 5 in graph, *Int. J. Appl. Math. Res.*, **4**(1) (2015), 30–51.

[5] S. K. Nechaev, M. V. Tamm, and O. V. Valba, Path counting on simple graphs: From escape to localization, *J. Stat. Mech. Theory Exp.* (2017), 053301. DOI10.1088/1742-5468/aa680a

[6] F. Harary and R. Norman, The dissimilarity characteristic of Husimi trees, *Ann. Math.*, **58** (1953), 134–141.

[7] F. Harary and E. M. Palmer, *Graphical enumeration*, Academic Press, 1973.

Received: 15 November 2024
Accepted: 15 June 2025

[8] F. Harary and G. E. Uhlenbeck, On the number of Husimi trees. *Proceedings of the National Academy of Sciences*, **39** (1953), 315–322.

[9] K. Husimi, Note on Mayer's theory of cluster integrals, *J. Chem. Phys.*, **18** (1950), 682–684.

[10] J. E. Mayer and M. G. Mayer, *J. Stat. Mech.*. John Wiley & Sons, 1940.

[11] R. J. Riddell, *Contribution to the theory of condensation*, Ph.D. thesis, University of Michigan, 1951.

[12] G. E. Uhlenbeck, Some basic problems of statistical mechanics. *American Mathematical Society*, Gibbs Lecture, 1950.

[13] H. S. Wilf, *generatingfunctionology* (3rd ed.). A K Peters/CRC Press, 2005.

Pennapa Chodok, Pitsinee Meeboon
Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi
pennapa.22544@mail.kmutt.ac.th, pitsinee.2243@mail.kmutt.ac.th

Pawaton Kaemawichanurat, Natawat Klamsakul
Mathematics and Statistics with Applications (MaSA),
Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi,
Bangkok, Thailand
pawaton.kae@kmutt.ac.th, natawat.kla@kmutt.ac.th