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When is the product of claw-free perfect graphs elementary?

Nandana K. Vasudevan and K. Somasundaram

Abstract. A graph G is said to be *perfect* if, for every induced subgraph H of G, we have $\chi(H) = \omega(H)$, where $\chi(H)$ and $\omega(H)$ are the chromatic number and the clique number of H, respectively. A graph is said to be *claw-free* if it does not have the complete bipartite graph $K_{1,3}$ as an induced subgraph. The class of claw-free perfect graphs is an important subclass of perfect graphs. A graph G is said to be an elementary graph if its edges can be colored with two colors such that every induced path of length two has its edges colored distinctly. In this paper, we answer the title question for the four standard graph products.

References

[1] Vašek Chvátal and Najiba Sbihi, Recognizing claw-free perfect graphs, *J. Combin. Theory Ser. B* **44**(2) (1988), 154–176.

[2] Sylvain Gravier, Frédéric Maffray, and Lucas Pastor, On the choosability of claw-free perfect graphs, *Graphs Combin.* **32** (2016), 2393–2413.

[3] Richard Hammack, Wilfried Imrich, and Sandi Klavžar, *Handbook of product graphs*, CRC Press, 2011.

[4] Frédéric Maffray and Bruce A Reed, A description of claw-free perfect graphs, *J. Combin. Theory Ser. B* **75**(1) (1999), 134–156.

[5] Gundgurthi Ravindra, Perfectness of normal products of graphs, *Discrete Math.* **24**(3) (1978), 291–298.

[6] Gundgurthi Ravindra and Kalyanapuram Rangachari Parthasarathy, Perfect product graphs, *Discrete Math.* **20** (1977), 177–186.

[7] Gert Sabidussi, Graphs with given group and given graph-theoretical properties, *Canadian J. Math.* **9** (1957), 515–525.

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Corresponding author: K. Somasundaram <s_sundaram@cb.amrita.edu>

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