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A note on one-hole domino tilings of squares and rectangles

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Abstract. We consider the number of domino tilings of an odd-by-odd rectangle that leave one hole. This problem is equivalent to the number of near-perfect matchings of the odd-by-odd rectangular grid. For any particular position of the vacancy on the $(2k+1) \times (2k+1)$ square grid, we show that the number of near-perfect matchings is a multiple of 2^k, and from this follows a conjecture of Kong that the total number of near-perfect matchings is a multiple of 2^k. We also determine the parity of the number of near-perfect matchings with a particular vacancy for the rectangle case.

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