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q-Analogues involving MacMahon's *q*-Catalan polynomials

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Abstract. Some terminal sum identities involving Catalan numbers have been introduced by Chu and Kılıç through the use of classical hypergeometric identities. In contrast to the q-analogues of these results that may be obtained with the use of ${}_r\phi_s$-series analogues in place of the classical identities applied by Chu and Kılıç, we obtain inequivalent q-Catalan identities through a discrete difference equations-based approach. These provide q-analogues of results from Chu and Kılıç and are obtained with the use of the q-difference case of Zeilberger's algorithm in conjunction with MacMahon's q-Catalan polynomials, and they are motivated by Andrews' results on q-analogues of Catalan number identities due to Touchard, Koshy, and Jonah. We also apply our results to introduce several Fibonomial sum identities.

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