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A unified treatment of some classic combinatorial inequalities using the variance method

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Abstract. The "variance method" has been used to prove many classical inequalities in design theory and coding theory. The purpose of this expository note is to review and present some of these inequalities in a unified setting. I also discuss some examples from my own research where I have employed these techniques.

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