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Counting Dyck paths with the long ascent

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Abstract. We study a family of Dyck paths characterized by the presence of long ascent, where the minimal ascent length is fixed. We derive an explicit enumeration formula for these paths and reveal connections to classical integer sequences, including partial sums of Mersenne numbers. A short bijective argument links these paths to subsets of integers, offering a clear combinatorial interpretation.

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