Cycles of length 4 or 8 in graphs with diameter 2 and minimum degree at least 3

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Abstract. In this short note it is shown that every graph of diameter 2 and minimum degree at least 3 contains a cycle of length 4 or 8. This result contributes to the study of the Erdős–Gyárfás Conjecture [1] by confirming it for the class of diameter-2 graphs.

References

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