BULLETIN OF THE ICA
Article in press.

The geoconvex number of a graph

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Abstract. For a graph G and vertices u and v, the interval $I(u, v)$ consists of u, v, and all vertices lying on some u-v geodesic in G. A subset $S \subseteq V(G)$ is said to be *convex* if, for every $u, v \in S$, the interval $I(u, v)$ is contained in S. A u-v geodesic is called a *convex geodesic* if $I(u, v)$ is convex, i.e., $I(I(u, v)) = I(u, v)$. A set $S \subseteq V(G)$ is called a *geoconvex set* if every vertex $x \in V(G)$ lies on some u-v convex geodesic in G, where $u, v \in S$. The minimum cardinality of a geoconvex set of G is called the *geoconvex number* of G, denoted by $g_{\mathrm{con}}(G)$. In this paper, we introduce a new parameter and establish a relationship between the geodetic number and the geoconvex number of a graph. We also characterize graphs for which $g_{\mathrm{con}}(G) = n$. Furthermore, we prove that for every triple of positive integers a, b, and n with $2 \leq a \leq b \leq n - 1$ there exists a connected graph of order n such that $g(G) = a$ and $g_{\mathrm{con}}(G) = b$.

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Key words and phrases: geodetic set, convex set, geodetic number, geoconvex number
Mathematics Subject Classifications: 05C12
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Received: 7 November 2025
Accepted: 4 April 2025

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