



# A new family of graceful spiders built from Rosa's path

N. B. HUAMANI AND M MUBEEN TAJUDEEN

**Abstract.** We prove that every spider  $T$  with  $7\ell - 1$  legs, of which  $3\ell$  have length  $4m + 1$  and  $4\ell - 1$  have length  $2m + 1$ , admits a graceful labeling. The construction is based on Rosa's classical graceful labeling of the path  $P_M$ , followed by a controlled cutting procedure and a reconnection at the center of the spider. For the family under consideration, this mechanism is encoded by an explicit labeling  $\Phi$ , obtained through a blockwise decomposition of  $P_M$ , a prescribed orientation pattern for the resulting components, and a reindexing of their vertices. We show that  $\Phi$  is injective and that the induced edge-label differences realize each integer from 1 to  $|E(T)|$  exactly once. This yields a new infinite family of graceful spiders whose number of legs grows linearly with  $\ell$  and whose leg lengths follow a rigid mixed pattern of the two odd values  $2m + 1$  and  $4m + 1$ .

## References

- [1] J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.* DS6 (2023), <https://doi.org/10.37236/27>.
- [2] A. Rosa, On certain valuations of the vertices of a graph, in *Theory of Graphs (International Symposium, Rome, July 1966)*, Gordon and Breach and Dunod, New York and Paris, (1967), 349–355.
- [3] N. B. Huamaní and M. J. Atoche, Families of graceful spiders with  $3\ell$ ,  $3\ell + 2$  and  $3\ell - 1$  legs, *Contemp. Math. (Singapore)* 5(4) (2024), 4908–4920, <https://doi.org/10.37256/cm.5420243289>.
- [4] A. Panpa, S. Imnang, and T. Wasuanankul, Graceful labeling of spider graphs with at most five legs, *J. Appl. Math.* 2025 (2025), Art. ID 5826777, <https://doi.org/10.1155/jama/5826777>.

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**Corresponding author:** Nelson Berrocal Huamani <[nelson.berrocal@unsch.edu.pe](mailto:nelson.berrocal@unsch.edu.pe)>