



A Discrete Logarithm Construction for Orthogonal Double Covers of the Complete Graph by Hamiltonian Paths

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Abstract. During their investigation of power-sequence terraces, Anderson and Preece briefly mention a construction of a terrace for the cyclic group $\mathbb{Z}_n$ when n is odd and $2n + 1$ is prime; it is built using the discrete logarithm modulo $2n + 1$. In this short note we see that this construction also gives a terrace when n is even and, in both cases, gives rise to an orthogonal double cover (ODC) for the complete graph K_n by Hamiltonian paths. This gives infinitely many new values for which such an ODC is known.

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