



Multicolor peg solitaire on graphs: Returning to the original configuration

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Abstract. Since 2011, there have been a number of papers that expand traditional peg solitaire for play on graphs. In a 2020 paper by Davis et al., the authors consider a variation in which the pegs take on one of two colors and the hole takes on a third. In this paper, we consider a generalization where the “colors” of the pegs come from the non-identity elements of a group Γ and the hole is labeled with the identity. Unlike most of the previous papers on peg solitaire, our interest is not in solving the graph. Instead, we consider the problem of when the vertices of a graph G can be labeled with elements of a group Γ and, after a sequence of peg solitaire moves, can be returned to the original configuration. If this is possible, then G is said to have a Γ -stability. In this paper, we provide simple necessary and sufficient conditions on a graph G and a group Γ so that G has a Γ -stability, where Γ is a finite group, an abelian group, or a free group. We also discuss the possibility where Γ is an infinite nonabelian group that is not a free group.

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