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Out-tournaments without disjoint cycles of different lengths

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Abstract. We show in this paper that every strong out-tournament with minimum out-degree 3, except the digraphs D_7^3 and D_8^3, contains two vertex-disjoint cycles of different lengths.

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