

2-switch: Transition and stability on forests and pseudoforests

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Abstract. Given any two forests (pseudoforests) with the same degree sequence, we show that one can be transformed into the other by a sequence of 2-switches in such a way that all the intermediate graphs of the transformation are forests (pseudoforests). We also prove that the 2-switch operation perturbs minimally some well-known integer parameters in families of graphs with the same degree sequence. Then, we apply these results to conclude that the studied parameters have the interval property in those families.

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