Chapter 4 Beyond Classical Search
4.1 Local search algorithms and optimization problems

CS4811 - Artificial Intelligence

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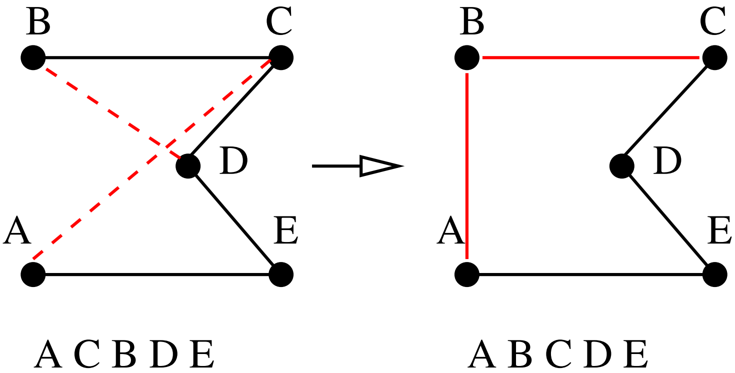
Outline

Iterative improvement algorithms

- In the problems we studied so far, the solution is the path. For example, the solution to the 8-puzzle is a series of movements for the "blank tile." The solution to the traveling in Romania problem is a sequence of cities to get to Bucharest.
- In many optimization problems, the path is irrelevant. The goal itself is the solution.
- The state space is set up as a set of "complete" configurations, the optimal configuration is one of them.
- An *iterative improvement algorithm* keeps a single "current" state and tries to improve it.
- The space complexity is constant!

Example: Travelling Salesperson Problem

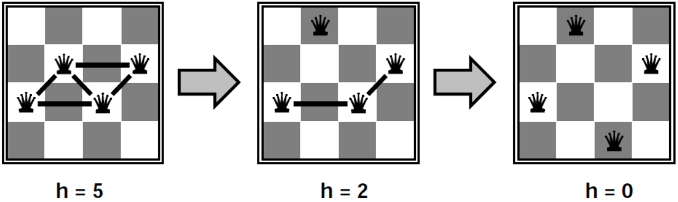
Start with any complete tour, perform pairwise exchanges



Variants of this approach get within 1% of optimal very quickly with thousands of cities.

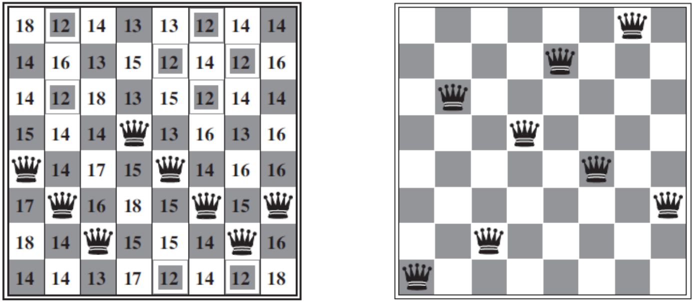
Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal.
Move a queen to reduce the number of conflicts (h).



Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1$ million.

Example: n-queens (cont'd)



(a) (b)

(a) shows the value of h for each possible successor obtained by moving a queen within its column. The marked squares show the best moves.
(b) shows a local minimum: the state has $h = 1$ but every successor has higher cost.

Hill-climbing (or gradient ascent/descent)

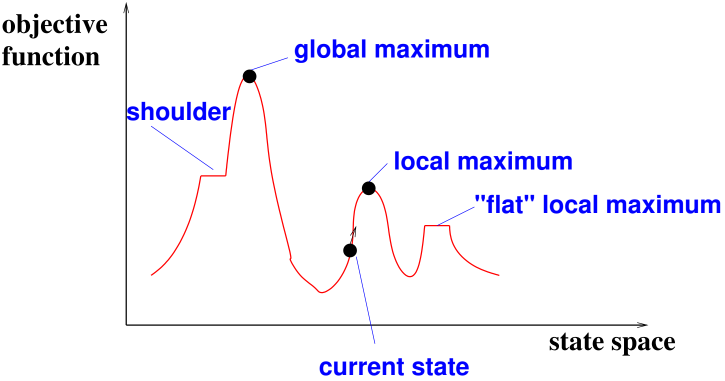
function HILL-CLIMBING (*problem*)
returns a state that is a local maximum

  inputs: *problem*, a problem
  local variables:
    current, a node
    neighbor, a node

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  loop do
    neighbor ← a highest-valued successor of *current*
    if *neighbor*.VALUE ≤ *current*.VALUE **then return** *current*.STATE
    current ← *neighbor*

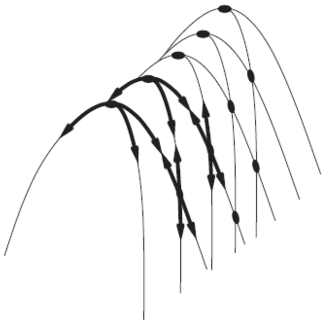
Hill-climbing (cont'd)

- "Like climbing Everest in thick fog with amnesia."
- Problem: depending on initial state, can get stuck on local maxima
- In continuous spaces, problems with choosing step size, slow convergence



Difficulties with ridges

The “ridge” creates a sequence of local maxima that are not directly connected to each other. From each local maximum, all the available actions point downhill.



Hill-climbing techniques

- stochastic: choose randomly from uphill moves
- first-choice: generate successors randomly one-by-one until one better than the current state is found
- random-restart: restart with a randomly generated initial state

Simulated annealing

function SIMULATED ANNEALING (*problem, schedule*)
returns a solution state

inputs:
problem, a problem
schedule, a mapping from time to "temperature"

local variables:
current, a node
next, a node

current $\leftarrow$ MAKE-NODE(*problem*.INITIAL-STATE)
for $t = 1$ **to** ∞ **do**
$T \leftarrow$ *schedule(t)*
if T=*0* **then return** *current*
next $\leftarrow$ a randomly selected successor of *current*
$\Delta E \leftarrow$ *next*.VALUE - *current*.VALUE
if $\Delta E > 0$ **then** *current* $\leftarrow$ *next*
else *current* $\leftarrow$ *next* only with probability $e^{\Delta E/T}$

Simulated annealing (cont'd)

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their size and frequency.
- Devised by Metropolis et al., 1953, for physical process modelling.
- At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

- When T is decreased slowly enough it always reaches the best state x^* because $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T. (Is this necessarily an interesting guarantee?)
- Widely used in VLSI layout, airline scheduling, etc.

Local beam search

- Idea: keep k states instead of 1; choose top k of all their successors
- Not the same as k searches run in parallel! Searches that find good states recruit other searches to join them.
- Problem: quite often, all k states end up on same local hill.
- To improve: choose k successors randomly, biased towards good ones.
- Observe the close analogy to natural selection!

The genetic algorithm

```
function GENETIC ALGORITHM (problem, FITNESS-FN)
returns an individual

    inputs:
        population, a set of individuals
        FITNESS-FN, a function that measures the fitness of an individual
    repeat
        new-population ← empty set
        for i = 1 to SIZE(population) do
            x ← RANDOM-SELECTION(population, FITNESS-FN)
            y ← RANDOM-SELECTION(population, FITNESS-FN)
            child ← REPRODUCE(x,y)
            if (small random probability) then child ← MUTATE(child)
            add child to new-population
        population ← new-population
    until some individual is fit enough, or enough time has elapsed
    return the best individual in population, according to FITNESS-FN
```

The crossover function

function REPRODUCE (x,y)
returns an individual

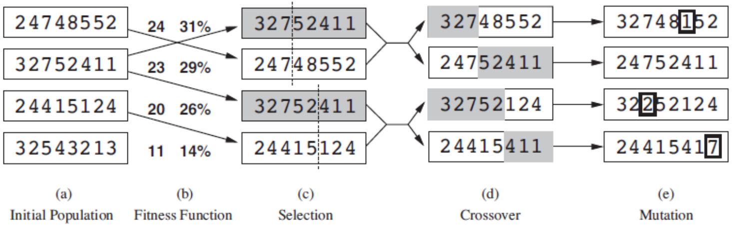
inputs:
x,y , parent individuals

$n \leftarrow \text{LENGTH}(x)$
$c \leftarrow$ random number from 1 to n
return $\text{APPEND}(\text{SUBSTRING}(x, 1, c), \text{SUBSTRING}(y, c + 1, n))$

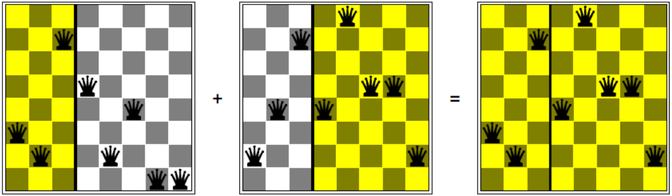
Genetic algorithms (GAs)

- Idea: stochastic local beam search + generate successors from pairs of states
- GAs require states encoded as strings.
- Crossover helps iff substrings are meaningful components.
- GAs $\neq$ evolution.
  e.g., real genes encode replication machinery.

Genetic algorithm example



The genetic algorithm with the 8-queens problem



Summary

- Hill climbing is a steady monotonous ascent to better nodes.
- Simulated annealing, local beam search, and genetic algorithms are “random” searches with a bias towards better nodes.
- All need very little space which is defined by the population size.
- None guarantees to find the globally optimal solution.

Sources for the slides

- AIMA textbook (3^{rd} edition)
- AIMA slides (http://aima.cs.berkeley.edu/)