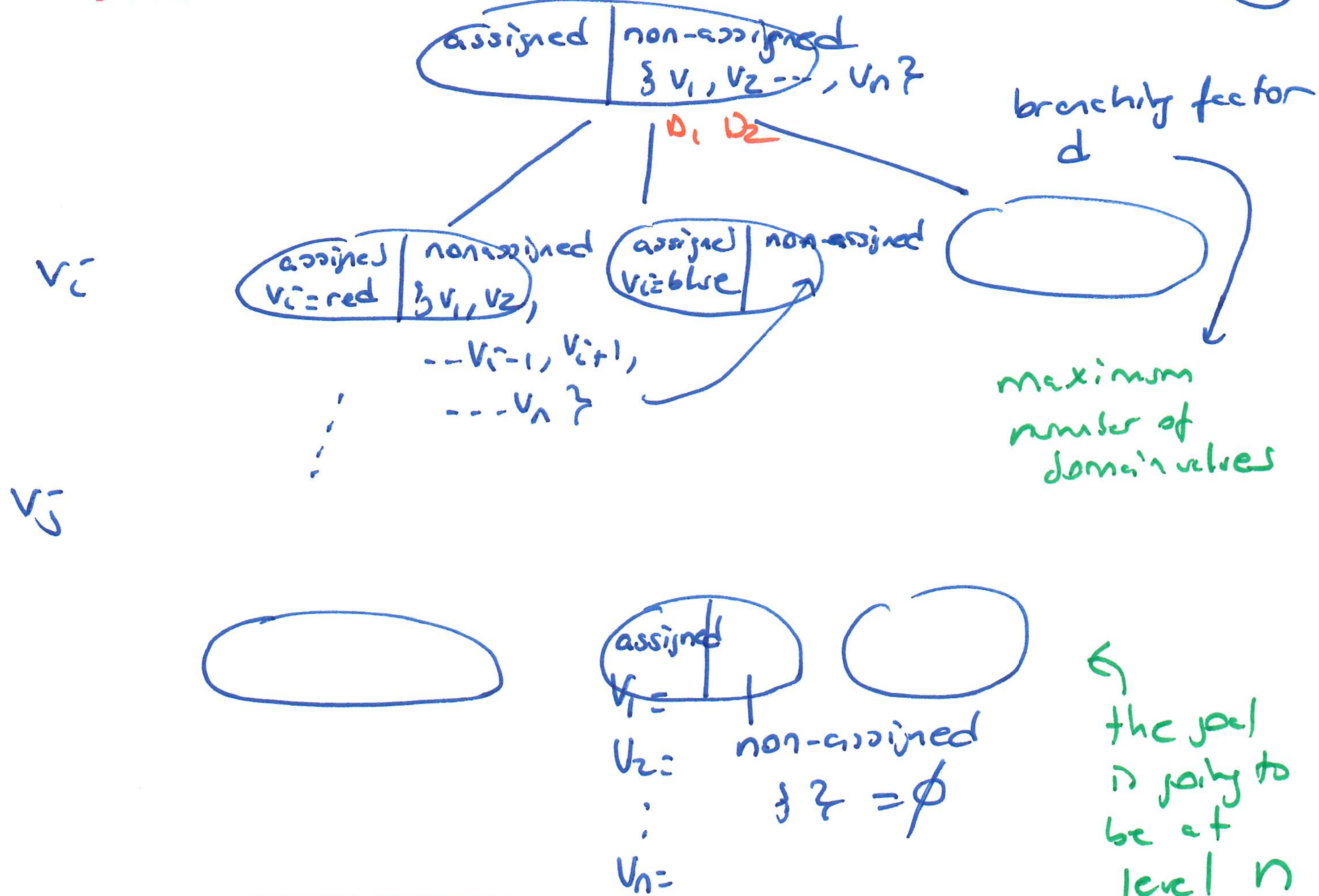
Previous class
- Constraint Satisfaction Problems (CSPs)
- Map coloring problem

Today
Continue with solving CSPs

A CSP is a "standardized" search problem.
problem definition → search process

A CSP:
- a set of variables $\{V_1, V_2, \ldots, V_n\}$
  (finite)
- for each variable: domain
  $D_1, D_2 \ldots D_n$
- set a constraints
  (finite)
  minimal case is binary constraints
  (between two variables)
  a unary constraint simply restricts
  the domain
- goal test
  - all the variables are assigned
  - all the constraints are satisfied

standard search process



plain DFS
will work fine
in most cases

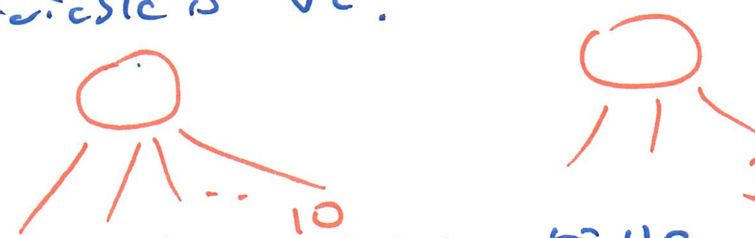
How to find the goal faster

$$f(n) = g(n) + h(n)$$

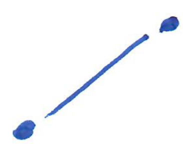
↗ not really an A^* style.

- which variable should be assigned next
  most constrained variable
  choose the variable with the fewest legal values

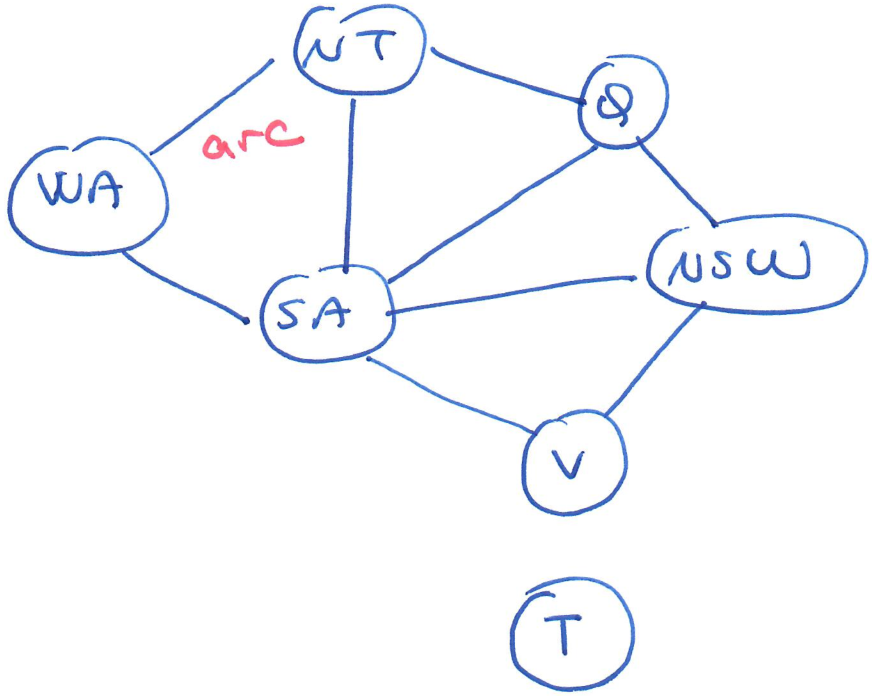
  which variable is V_i?



tiebreaker: most constraining variable

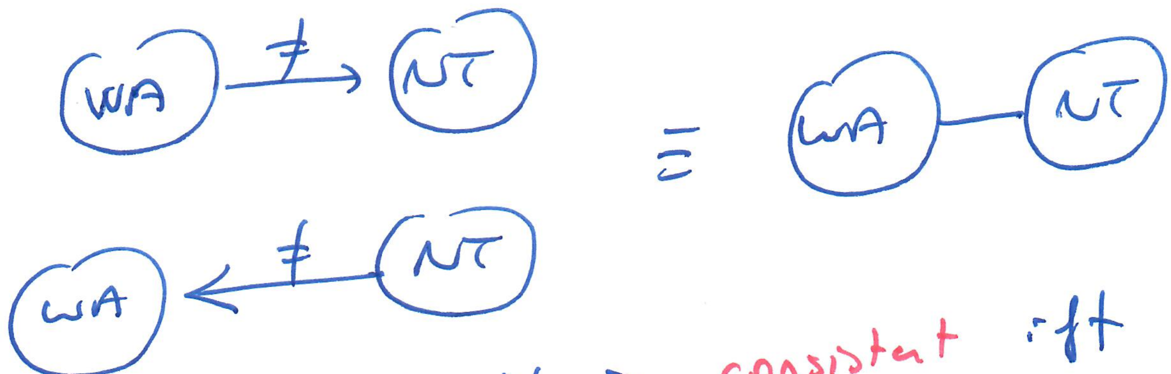


- least constraining value
  value that rules out fewest values among remaining variables
  (value that enables more values among remaining variables)
- forward checking
  we reduce the domains of the variables that have not been assigned yet.



a constraint graph

$a + b = c$

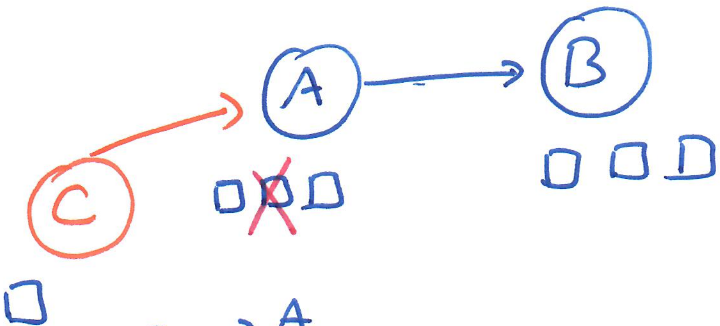


An arc (directed) $X \rightarrow Y$ is consistent iff for every value of X there is a possible value in Y's domain.

$V_1 - V_2$

$V_1 \rightarrow V_2$
$V_2 \rightarrow V_1$
⋮

} check all of them for <u>arc consistency</u>



$C \rightarrow A$

Time complexity of arc consistency

~~constraint graph~~ binary constraint graph with n nodes

how many arcs (edges) can we have?

not more than a fully connected graph

$O(n^2)$

d values in their domain

$n^2 \left\{ \begin{array}{l} V_1 \rightarrow V_2 \\ V_2 \rightarrow V_1 \\ \vdots \\ V_{n-1} \rightarrow V_n \end{array} \right.$

for each arc: n^2

check consistency

$V_i \rightarrow V_j$

↓

foreach value d

anything in V_j's domain?

~~$d-1$~~ d

V_i	V_j
r, g, b	r, g, b
d	d

so far:

cost of arc consistency

$n^2 d^2$

addition of repeated arc consistency checks.