Previous class

Representation: joint probability distribution table
p.d.

each row is a unique combination of the variables (world, discrete)

Reasoning: map the question to the rows of the joint p.d. table.

what is the probability of --- ?

$P(x)$

$\mathbb{P}(X)$

$P(x, y, z) \rightarrow$ mapping done through Boolean logic

$= P(x \land y \land z)$

$P(r, a, y) \rightarrow$ maps to one row

$P(r, a) \rightarrow$ maps to multiple rows

$P(r, a, y) + P(r, a, n)$

$\sum_{value_H} P(r, a, value_H) =$

H

These are prior probabilities

$P(r \rightarrow a) = P(\neg r \lor a)$

Posterior (conditional) probabilities

Something happened (the value of a variable is known)

evidence variable

$P(a|b)$ ← (focus only on the rows where b is true

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

$\hookrightarrow P(b \wedge a) + P(b \wedge \neg a)$

$P(a \wedge b) = P(a|b)\,P(b)$ product rule

$\mathbb{P}(A, B) = \mathbb{P}(A|B)\,\mathbb{P}(B)$

$P(a|b)$ $\quad P(b)$

$P(a|\neg b)$ $\quad P(\neg b)$

$P(\neg a|b)$

$P(\neg a|\neg b)$

$\langle P(a,b),\ P(a,\neg b) \qquad \rangle$

how many entries in this

4: 2 for A, 2 for B

$P(a|b)\,P(b)$

$P(a|\neg b)\,P(\neg b)$

$[\qquad][\qquad]$

Chain rule

$$p(a,b,c,d) =$$

$$= p(d \mid a,b,c)\, \underbrace{p(a,b,c)}_{p(c \mid a,b)\, \underbrace{p(a,b)}_{p(b \mid a)\, p(a)}}$$

$$p(a,b,c,d) = p(a)\, p(b \mid a)\, p(c \mid a,b)\, p(d \mid a,b,c)$$

$$x_1 \; x_2 \; x_3 \; x_4$$

$$p(x_1)\; p(x_2 \mid x_1)\, p(x_3 \mid x_1, x_2)$$

$$p(x_4 \mid x_1, x_2, x_3)$$

$$= \prod_{i=1}^{n} p(x_i \mid x_1 \ldots x_{i-1})$$

$$p(a,b,c,d) = p(a \mid b,c,d)\; p(b,c,d)$$

$$p(b \mid c,d)\, p(c,d)$$

$$\mathbb{P}(Cavity \mid toothache)$$

$$= \frac{\mathbb{P}(Cavity, toothache)}{P(toothache)}$$

$$= \left\langle \frac{P(cavity, toothache)}{P(toothache)}, \frac{P(\neg cavity, toothache)}{P(toothache)} \right\rangle$$

$$= \alpha \langle P(cavity, toothache), P(\neg cavity, toothache) \rangle$$

$$\mathbb{P}(Cavity \mid toothache) =$$

$$= \alpha \, \mathbb{P}(Cavity, toothache)$$