Previous class

Representation: joint probability distribution table

P.D.

each row is a unique configuration of the vehicles (world, discrete)

Reasoning: map the question to the rows of the joint p.d. table.

What is the probability of \( \cdot \cdot \cdot \) ?

\[ P(x) \]

\[ P(X) \]

\[ P(x, y, z) \rightarrow \text{mapping done through Boolean logic} \]

\[ P(x \land y \land z) \]

\[ P(r, a, y) \rightarrow \text{maps to one row} \]

\[ P(r, a) \rightarrow \text{maps to multiple rows} \]

\[ P(r, a, y) + P(r, a, y) \leq \]

\[ \sum_{\text{value}_H} P(r, a, \text{value}_H) = H \]

These are prior probabilities

\[ P(r \rightarrow a) = P(r \rightarrow \text{Va}) \]
Posterior (conditional) probabilities

Something happened (the value of a variable is known)

evidence variable

\[ P(a|b) \]

focus only on the rows

where \( b \) is true

\[ P(a|b) = \frac{P(a \land b)}{P(b)} \]

\[ \Rightarrow P(a \land b) = P(a|b) P(b) \]

product rule

\[ P(A, B) = \frac{P(A \land B) P(B)}{P(a \land b)} \]

\[ \Rightarrow \begin{cases} \frac{P(a \land b)}{P(a \land b) P(b) P(c \land b)} & \text{has many entries in this} \\ 4: 2 \text{ for } A, 2 \text{ for } B \\ \end{cases} \]
Chain rule

\[
\rho(a, b, c, d) = \rho(c | a, b, c) \rho(a, b, c)
\]

\[
= \rho(c | a, b) \rho(a, b)
\]

\[
\rho(c | a, b) \rho(a, b)
\]

\[
\rho(b | a) \rho(a)
\]

\[
\rho(a, b, c, d) = \rho(a) \rho(b | a) \rho(c | a, b) \rho(d | a, b, c)
\]

\[
\rho(x_i) \rho(x_2 | x_1) \rho(x_3 | x_1, x_2)
\]

\[
\rho(x_4 | x_1, x_2, x_3)
\]

\[
= \prod_{i=1}^{n} \rho(x_i | x_1 \ldots x_{i-1})
\]

\[
\rho(a, b, c, d) = \rho(a | b, c, d) \rho(b, c, d)
\]

\[
\rho(b | c, d) \rho(c, d)
\]
\[ P(\text{Cavity} | \text{Toothache}) = \frac{P(\text{Cavity, Toothache})}{P(\text{Toothache})} \]

\[ = \langle \frac{P(\text{Cavity, Toothache})}{P(\text{Toothache})}, \frac{P(\neg \text{Cavity, Toothache})}{P(\text{Toothache})} \rangle \]

\[ = \alpha \langle P(\text{Cavity, Toothache}), P(\neg \text{Cavity, Toothache}) \rangle \]

\[ P(\text{Cavity} | \text{Toothache}) = \alpha \frac{P(\text{Cavity, Toothache})}{P(\text{Toothache})} \]