Previous class

- Conditional probability

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \wedge B)}{\mathbb{P}(B)}$$

- Product rule

$$P(a \wedge b) = P(a|b)\, P(b)$$

$$\mathbb{P}(A \wedge B) = \mathbb{P}(A|B)\, \mathbb{P}(B)$$

- Chain rule is a generalization of product rule

$$P(a,b,c,d) = P(a|b,c,d)\, P(b|c,d)\, P(c|d)\, P(d)$$

$$\mathbb{P}(A,B,C,D) = \mathbb{P}(A|B,C,D)\, \mathbb{P}(B|C,D)\, \mathbb{P}(C|D)\, \mathbb{P}(D)$$

- Independence

$\mathbb{P}(A|B) = \mathbb{P}(A)$ or $\mathbb{P}(B|A) = \mathbb{P}(B)$ or $\mathbb{P}(A,B) = \mathbb{P}(A)\mathbb{P}(B)$

"A and B are independent."

- Conditional independence

$$\mathbb{P}(A|B,C) = \mathbb{P}(A|C)$$

"A and B are conditionally independent given C"

$$\mathbb{P}(A,B,C,D,E,F)$$

$$\mathbb{P}(A|B,C,D,E,F,G)$$

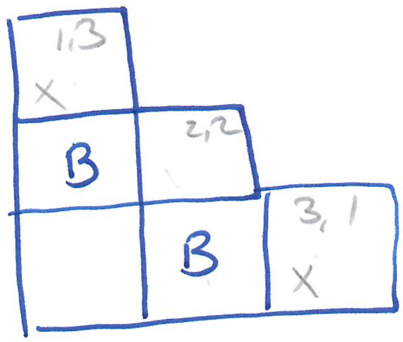
$$= \mathbb{P}(A|G)$$

If we know that A and (B, C, D, E, F) are conditionally independent given C we need to enumerate 1 rather than 6.

The wumpus world

A pit causes a breeze in the adjacent cells. (domain knowledge)

The wumpus causes a stench in the adjacent cells. (domain knowledge)



at 2,2 only

$0.2 \times 0.8 \times 0.8$

at 1,3 and 3,1 are not possible

all three

$0.2 \times 0.2 \times 0.2$

at 1,3 and 3,1

$0.8 \times 0.2 \times 0.2$

Full joint distribution

$$\mathbf{P}(P_{1,1}, \ldots, P_{4,4}, B_{1,1}, B_{1,2}, B_{2,1}) =$$

$$\mathbf{P}(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,1} \ldots P_{4,4})$$

$$\times$$

$$\mathbf{P}(P_{1,1} \ldots P_{4,4})$$

observations (evidence)

breezes — pits

$b = \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$

$known = \neg P_{1,1} \wedge \neg P_{1,2} \wedge \neg P_{2,1}$

query $\mathbb{P}(P_{1,3} \mid known, b)$

$\mathbb{P}(P_{2,2} \mid known, b)$

$\mathbb{P}(P_{3,1} \mid known, b)$

$\mathbb{P}(P_{1,3} \mid known, b) =$

sum over the hidden variables pits at other locations

$\alpha\, \mathbb{P}(P_{1,3}, known, b)$

total 16 cells

known 3 cells

query 1 cell cell (1,3)

$16-3-1=12$ "unknown" cells

$$\sum_{unknown} \mathbb{P}(P_{1,3} \mid known, b, unknown)$$

$$= \alpha \sum_{unknown} \mathbb{P}(P_{1,3}, known, b, unknown)$$

$\mathbb{P}(b \mid P_{1,3}, known, unknown)$

$= \mathbb{P}(b \mid P_{1,3}, known, frontier, other)$

$= \mathbb{P}(b \mid P_{1,3}, known, frontier)$

$=$

$\mathbb{P}(P_{1,3} \mid known, b)$

$$= \frac{\mathbb{P}(P_{1,3}, known, b)}{P(known, b)}$$

same denominator in the distribution so can be leaded to α

$= \alpha \, \mathbb{P}(P_{1,3}, known, b)$

put the hidden variables

$= \alpha \sum_{unknown} \mathbb{P}(P_{1,3}, known, b, unknown)$

apply the chain rule

$= \alpha \sum_{unknown} \mathbb{P}(b \mid P_{1,3}, known, unknown) \times \mathbb{P}(P_{1,3}, known, unknown)$

$unknown = frontier \cup other$

$= \alpha \sum_{frontier} \sum_{other} \mathbb{P}(b \mid P_{1,3}, known, \cancel{unknown}\ frontier, other) \times \mathbb{P}(P_{1,3}, known, unknown)$