CS5811
October 14, 2020
Wednesday ①

Reminder

Exam 1 is next Wednesday.
October 21, 2020 6:00 pm
Covers: Chapter 3 : Search
Chapter 6 : CSPs
\+ : Temporal CSPs (PA, IA, TCSP)
STN

②

The Wumpus World: Derivation of the Probability Formula

Define $unknown = frontier \cup other$

$\mathbb{P}(b|P_{1,3}, known, unknown) \quad = \mathbb{P}(b|P_{1,3}, known, frontier, other) \quad = \mathbb{P}(b|P_{1,3}, known, frontier)$

We want to manipulate the query into a form where we can use the above conditional independence.

This is what we would like to compute: $\quad \mathbb{P}(P_{1,3}|known, b)$

$\alpha < P_{1,3} , \neg P_{1,3} >$

Use the definition of conditional probability:
$= \mathbb{P}(P_{1,3}, known, b)/P(known, b)$

$P(a,b,c,d)$

$P(a|b,c,d)$

$P(r,a) — P(r,a,y) \; P(b,c,d)$

$H \; \overset{yes}{\underset{no}{}} \; + \; P(r,an)$

The denominators of the two instantiations of $P_{1,3}$ will be the same.
Thus, we can use the normalizing constant α, which will be $1/P(known, b)$:
$= \alpha \; \mathbb{P}(P_{1,3}, known, b)$

Sum over the hidden variables:
$= \alpha \sum_{unknown} \mathbb{P}(P_{1,3}, known, b, unknown)$

Pits cause a breeze at the adjacent cells. Use the chain rule so that the causes are the evidence.
$= \alpha \sum_{unknown} \mathbb{P}(b|P_{1,3}, known, unknown) \; \mathbb{P}(P_{1,3}, known, unknown)$

Write $unknown$ as $frontier \cup other$:
$= \alpha \sum_{frontier} \sum_{other} \mathbb{P}(b|P_{1,3}, known, frontier, \cancel{other}) \; \mathbb{P}(P_{1,3}, known, frontier, other)$

Use the conditional independence of b and $other$ given $frontier$:
$= \alpha \sum_{frontier} \sum_{other} \mathbb{P}(b|P_{1,3}, known, frontier) \; \mathbb{P}(P_{1,3}, known, frontier, other)$

Move $\sum_{other}$ inward:
$= \alpha \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) \sum_{other} \mathbb{P}(P_{1,3}, known, frontier, other)$

The probability of a pit at a cell is independent of the others ($P_{i,j} = 0.2$ for all i and j):
$= \alpha \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) \sum_{other} \mathbb{P}(P_{1,3}) P(known) P(frontier) P(other)$

Move $P(known)$ and $\mathbb{P}(P_{1,3})$ outside of the summations:
$= \alpha \; P(known) \mathbb{P}(P_{1,3}) \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) \sum_{other} P(frontier) P(other)$

Fold $P(known)$ into the normalizing constant α:
$= \alpha' \; \mathbb{P}(P_{1,3}) \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) \sum_{other} P(frontier) P(other)$

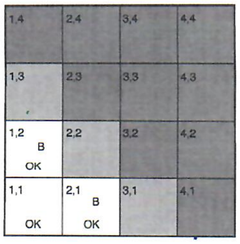
Move $\sum_{other}$ inward again:
$= \alpha' \; \mathbb{P}(P_{1,3}) \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) P(frontier) \sum_{other} P(other)$

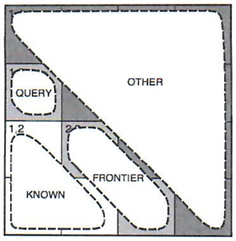
Consider $\sum_{other} P(other)$. Summation over all possibilities of $other$, $P(other)$ is 1:
$= \alpha' \; \mathbb{P}(P_{1,3}) \sum_{frontier} \mathbb{P}(b|P_{1,3}, known, frontier) P(frontier)$

Details of the wumpus world calculations

$\langle \quad \rangle$

$P_{1,3}, \neg P_{1,3}$





$0.8 \quad 0.8 \quad 0.8$

$P(\neg P_{1,1}, \neg P_{1,2}, \neg P_{2,1})$

We know the following facts (evidence): $b = \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$ $known = \neg p_{1,1} \wedge \neg p_{1,2} \wedge \neg p_{2,1}$

$\mathbb{P}(P_{1,3} | \text{ known, b}) = \alpha' \, \mathbb{P}(P_{1,3}) \sum_{frontier} \mathbb{P}(\text{ b } | \text{ known, } P_{1,3}, \text{ frontier}) \text{ P(frontier)}$

why is P(known) a constant?

First, compute with $P_{1,3}$ = true:

Compute the sum over the frontier:

4 possibilities

frontier
2,2
3,1

$\text{P(b } | \text{ known, } P_{1,3}, P_{2,2}, P_{3,1}) \text{ P}(P_{2,2}, P_{3,1}) = 1 \times 0.2 \times 0.2 = 0.04$
$\text{P(b } | \text{ known, } P_{1,3}, \neg P_{2,2}, P_{3,1}) \text{ P}(\neg P_{2,2}, P_{3,1}) = 1 \times 0.8 \times 0.2 = 0.16$
$\text{P(b } | \text{ known, } P_{1,3}, P_{2,2}, \neg P_{3,1}) \text{ P}(P_{2,2}, \neg P_{3,1}) = 1 \times 0.2 \times 0.8 = 0.16$
$\text{P(b } | \text{ known, } P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) \text{ P}(\neg P_{2,2}, \neg P_{3,1}) = 0 \times 0.8 \times 0.8 = 0.00$

The sum is: 0.04 + 0.16 + 0.16 + 0.00 = 0.36. $\text{P}(P_{1,3}) = 0.2$, therefore:

$\text{P}(P_{1,3}) \sum_{frontier} \text{P(b } | \text{ known, } P_{1,3}, \text{ frontier) P(frontier)} = 0.2 \times 0.36 = \mathbf{0.072}.$

Then, compute with $P_{1,3}$ = false:

Compute the sum over the frontier:

$\text{P(b } | \text{ known, } \neg P_{1,3}, P_{2,2}, P_{3,1}) \text{ P}(P_{2,2}, P_{3,1}) = 1 \times 0.2 \times 0.2 = 0.04$
$\text{P(b } | \text{ known, } \neg P_{1,3}, \neg P_{2,2}, P_{3,1}) \text{ P}(\neg P_{2,2}, P_{3,1}) = 0 \times 0.8 \times 0.2 = 0.00$
$\text{P(b } | \text{ known, } \neg P_{1,3}, P_{2,2}, \neg P_{3,1}) \text{ P}(P_{2,2}, \neg P_{3,1}) = 1 \times 0.2 \times 0.8 = 0.16$
$\text{P(b } | \text{ known, } \neg P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) \text{ P}(\neg P_{2,2}, \neg P_{3,1}) = 0 \times 0.8 \times 0.8 = 0.00$

The sum is: 0.04 + 0.00 + 0.16 + 0.00 = 0.20. $\text{P}(\neg P_{1,3}) = 0.8$, therefore:

$\text{P}(P_{1,3}) \sum_{frontier} \text{P(b } | \text{ known, } P_{1,3}, \text{ frontier) P(frontier)} = 0.8 \times 0.20 = \mathbf{0.16}.$

$\mathbb{P}(P_{1,3} | \text{ known, b}) = \alpha' < \mathbf{0.072, 0.16} > = < \frac{0.072}{0.072+0.16}, \frac{0.16}{0.072+0.16} > = < 0.31, 0.69 >$

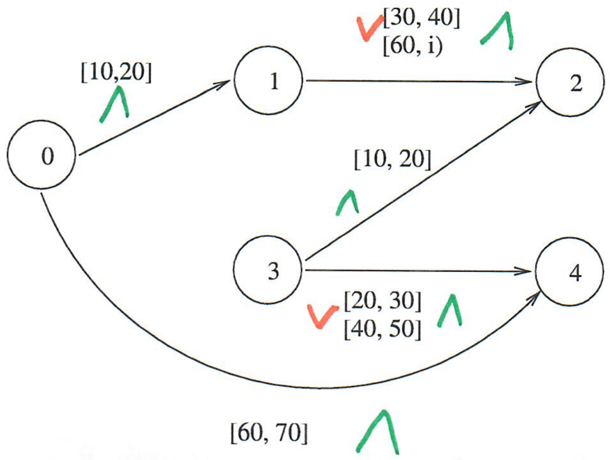
$\underline{\text{P}(P_{1,3} | \text{ known, b}) = 0.31}$ (= $\text{P}(P_{3,1} |$ known, b) by symmetry)

CS 5811 Homework 2: Point Algebra and Quantitative Temporal Networks

Due: Wednesday, October 14 , 2020, 10:00am (in the morning, 1 hour before class)
Submission: Type your answer and submit a pdf file. Scanned submissions are not allowed.
You must cite every source that you use for each question. Include all the sources you have consulted even if you didn't write from them.
If you developed your own solution, you must write that you haven't looked at any solutions or related material.

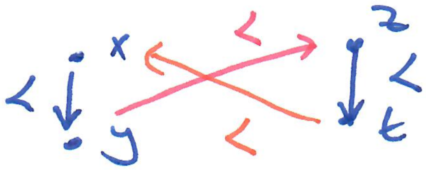
Question 1. *(20 points)* On TCSP slide 24, it is stated that $I\{b, a\}J$ where $I = [x, y]$ and $J = [z, t]$ cannot be represented with a PA network. Draw a PA network that attempts to represent I is before or after J and explain why the representation doesn't work.

Question 2. *(40 points)* Show that TCSP Slide 30's scenario where John takes the car and Fred takes the carpool is consistent. Show that the alternate scenario, in which John used a bus and Fred used a carpool is not consistent.



Question 3. *(40 points)* Represent the following problem using TCSPs. Clearly show what the events are. Convert the TCSPs into one or more STNs. Solve the STN(s) using the d-graph and all-pairs-shortest-path technique and explain the result.

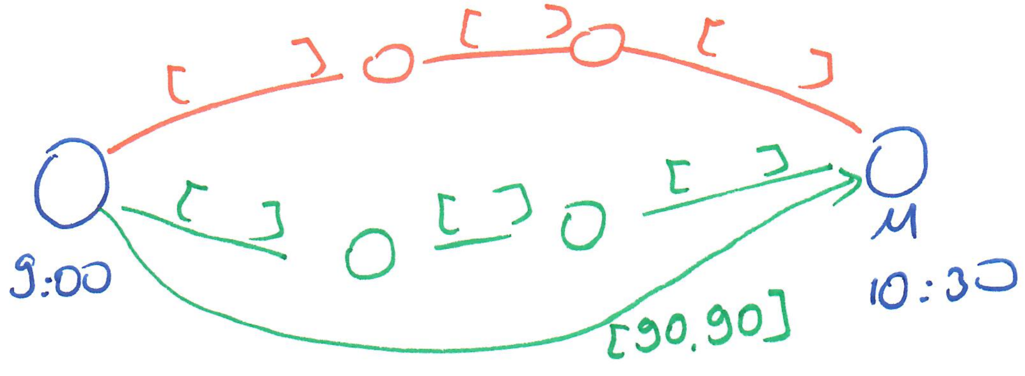
> It is now 9:00am. The remote sensing station has an important maintenance scheduled at 10:30. Before the maintenance, one of the two experiments should be performed. The scientists can either use the Pierre Auger fluorescence detector to detect cosmic rays or use the photometric telescope for a survey. Each experiment takes 30 minutes. The absolute calibration of the Pierre Auger fluorescence detector relates the flux of photons of a given wavelength at the detector aperture to the electronic signal recorded by the FD data acquisition system, and takes 65-75 minutes. The photometric telescope is self-calibrating but needs to cool down for 30-40 minutes before being shut down for maintenance. Which experiment should the scientists work on?

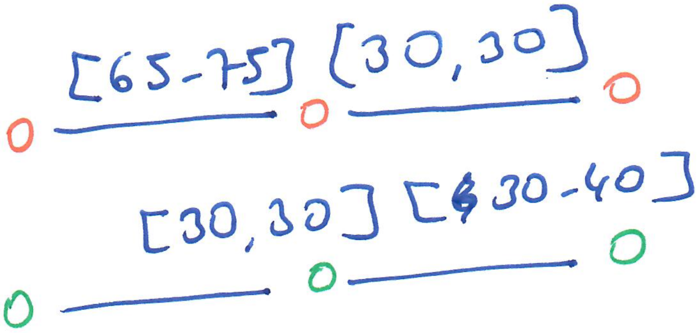


$x < y < z < t$

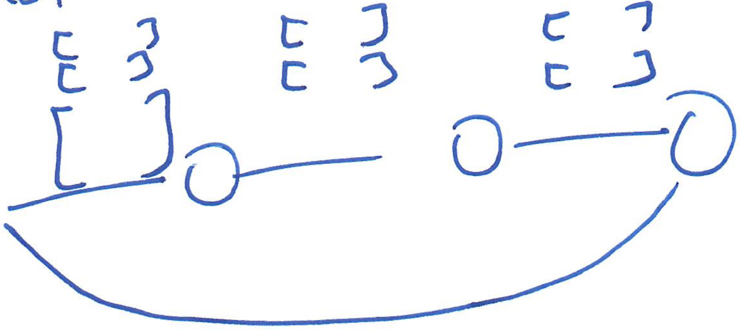
$x < t$

$t < x$





Does not contain "or"s.



[30,30) [30,40]