CS5811
Nov. 2, 2020
Monday
(1)

Reminder

IAAI presentations next week
(attendance required)
(participation required)

Previous class

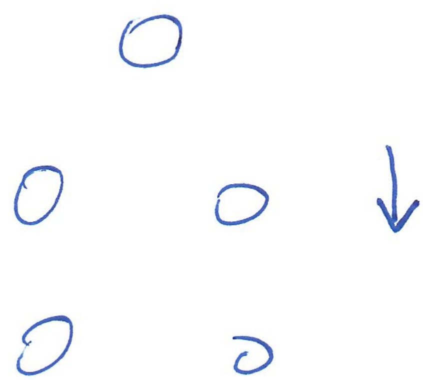
BBNs: approximate inference
- rejection sampling
- likelihood weighting

MCMC ($\underbrace{\text{Markov}}_{\text{no history}}$ Chain $\underbrace{\text{Monte Carlo}}_{\text{non deterministic}}$) simulation

$$\begin{array}{cccccc} [\; t & \boxed{t} & \boxed{f} & t & f \;] \\ x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

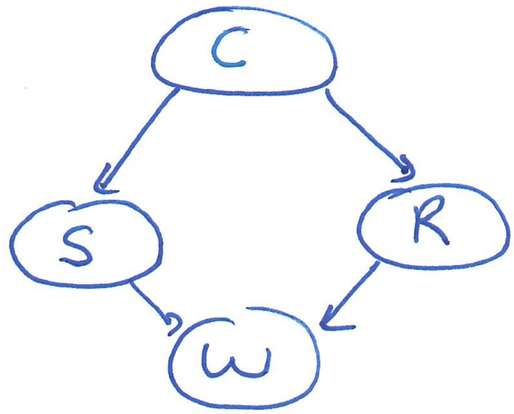
will use Markov blanket

[m _ m _ _]



$P(x|e)$

$w =$



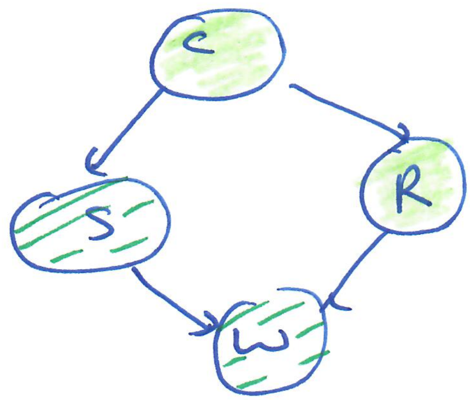
Query

$\mathbb{P}(Rain \mid s, w)$

sprinkler = true

wetgrass = true

Set the evidence variables



Initialize C and R randomly

Determine an order to sample the variables

C R

Sample C : what probability distribution are we going to use?

$\mathbb{P}(C \mid mb(C))$

$$P(x_i \mid mb(X_i)) = \alpha\, P(x_i \mid parents(X_i)) \prod_{y_j \in children(X_i)} P(y_j \mid parent(y_j))$$

parents
children
children's parents

child of X_i

$\mathbb{P}(C \mid mb(C)) = \alpha \mathbb{P}(C) \underbrace{\mid \quad)}_{\text{parents of } C}$ x

none

$P(s \mid C) \times P(r \mid C)$

$\alpha \langle \underset{C=t}{0.5}, \underset{C=f}{0.5} \rangle \langle \underset{C=t}{0.1}, \underset{C=f}{0.5} \rangle \langle \underset{C=t}{0.8}, \underset{C=f}{0.2} \rangle$

$\alpha \langle \underset{C=t}{0.5 \times 0.1 \times 0.8}, \underset{C=f}{0.5 \times 0.5 \times 0.2} \rangle$

$5 \times 8 \times 10^{-3}$ $\quad$ $5 \times 5 \times 2 \times 10^{-3}$

0.04 $\quad$ 50×10^{-3}

$\alpha \langle 0.04, 0.05 \rangle$

we found the probability distribution to sample C from $\langle 0.04, 0.05 \rangle$

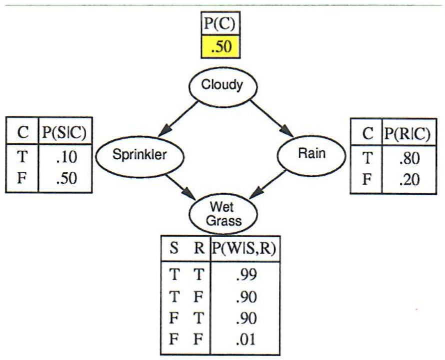
C	S	R	W
(t	t	t	t)

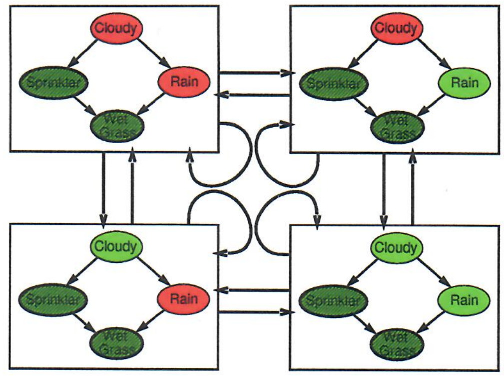
let's say we get C=f our new sample will be

(f	t	t	t)

sample : R to sample R compute $\mathbb{P}(R \mid mb(R))$ $\langle \; , \; \rangle$

CS5811 handout - Markov Chain Monte Carlo (MCMC) sampling
The procedure to obtain the sampling distributions





The probability of a variable given its Markov blanket is proportional to the probability of the variable given its parents times the probability of each child given its respective parents:

$$P(x_i|mb(X_i)) = \alpha P(x_i|parents(X_i) \times \prod_{Y_j \in children(X_i)} P(y_j|parents(Y_j))$$

Consider the query $\mathbb{P}(R|S = t, W = t)$.
S is true from the evidence. Suppose that R is true in the state.
We will be sampling for C.

The Markov blanket of C is its parents ($\emptyset$), its children ($\{R, S\}$), and the other parents of its children ($\emptyset$). We use the following distributions to sample C.

$\mathbb{P}(C|mb(C)) = \mathbb{P}(C|R = t, S = t) = \alpha\ \mathbb{P}(C)\ \mathbb{P}(S = t|C)\ \mathbb{P}(R = t|C)$
$= \alpha < 0.5, 0.5 > < 0.1, 0.5 > < 0.8, 0.2 >$
$= \alpha < 0.04, 0.05 >$
$=< \frac{4}{9}, \frac{5}{9} >$

For the states where R is false, $\mathbf{P}\ (C|\neg R, S)$ is calculated similarly .

S is true from the evidence. Suppose that C is true in the state.
We will be sampling for R.

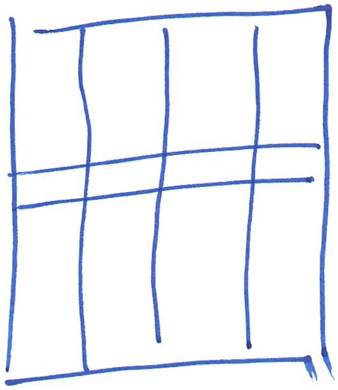
The Markov blanket of R is its parents ($\{C\}$), its children ($\{W\}$), and the other parents of its children ($\{S\}$). We use the following distributions to sample R.

$\mathbb{P}(R|mb(R)) = \mathbb{P}(R|C = t, S = t, W = t) = \alpha\ \mathbb{P}(R|C = t)\ \mathbb{P}(W = t|R, S = t)$
$= \alpha < 0.8, 0.2 > < 0.99, 0.90 >$
$= \alpha < 0.792, 0.18 >$
$= \alpha < \frac{0.792}{0.972}, \frac{0.18}{0.972} > < \frac{22}{27}, \frac{5}{27} >$

$\mathbb{P}(R|\neg C, S, W)$ is calculated similarly.

wrap-up Chapter 14

domain n variables (Boolean)



we need a table with n columns and

$\underbrace{2^n \text{ rows}}$ ($2^n - 1$)

hard to represent

hard to provide numbers

compact representation of the full joint probability distribution table (jpd)

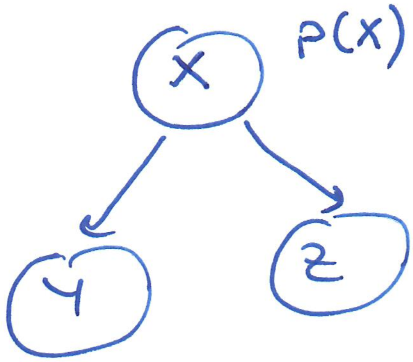
of all of the jpd table

(an approximation)

BBN

directed, acyclic graph

$P(X)$



$P(z \mid x)$

$P(z \mid \neg x)$

} conditional probability distribution table (cpt) exact inference

$$P(x_1, \ldots x_n) = P(x_n | x_1 \ldots x_{n-1})$$

remove the non parents from here

$$\times\ P(x_{n-1} | x_1 \ldots x_{n-2})$$

$$\vdots$$

$$\times\ P(x_2 | x_1)$$

$$P(x_1)$$

read the values from cpt's

$$\mathbb{P}(X|e) = \sum_h (X|e,h)$$

left with k variables

exponential $\rightarrow 2^k$

Solution: approximate inference

- rejection sampling
  may discard too many samples
- likelihood weighting
  samples with very very small weights
- MCMC Gibbs