Chapter 3 Solving Problems by Searching
3.5 –3.6 Informed (heuristic) search strategies
More on heuristics

CS5811 - Advanced Artificial Intelligence

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A^* search

We have seen that A^* search

- May have exponential time and space complexity but will perform well with a good heuristic
- Is complete
- Finds the optimal solution

We will look at another property that affects how the search proceeds.

Consistency

A heuristic is *consistent* if

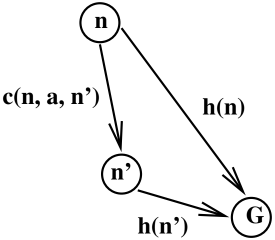
$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

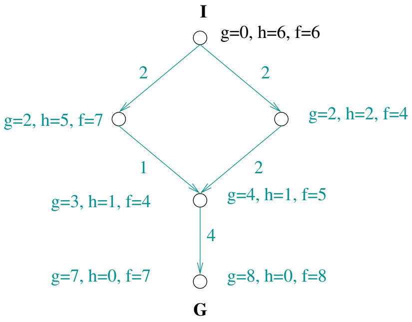
$$\begin{aligned} f(n') &= \text{g(n') + h(n')} \\ &= \text{g(n) + c(n,a,n') + h(n')} \\ &\geq \text{g(n) + h(n)} \\ &= \text{f(n)} \end{aligned}$$

We get $f(n') \geq f(n)$, i.e.,
$f(n)$ is nondecreasing along any path.

Consistency is the *triangle inequality* for heuristics.

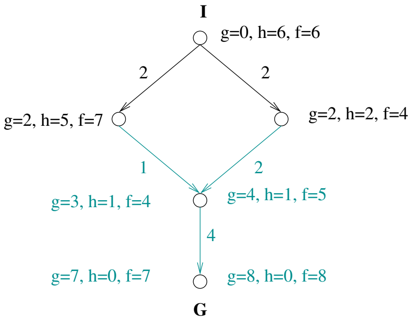


Progress of A* with an inconsistent heuristic



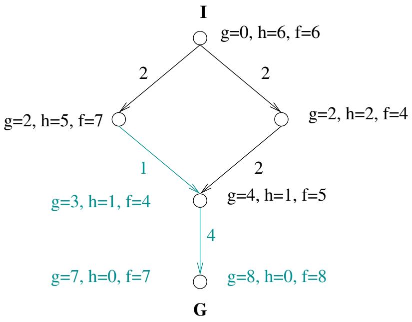
Note that *h* is admissible.
It never overestimates.

Progress of A* with an inconsistent heuristic



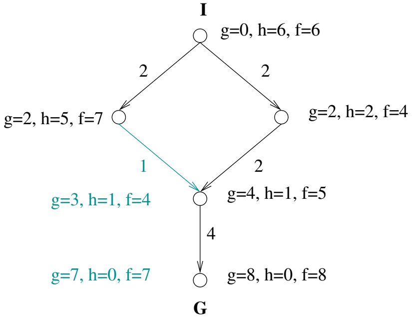
The root node was expanded.
Note that f decreased from 6 to 4.

Progress of A* with an inconsistent heuristic



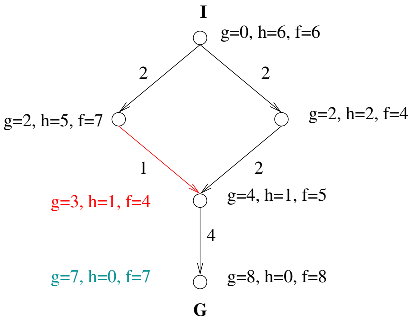
The suboptimal path is being pursued.
The right hand side path is suboptimal.

Progress of A* with an inconsistent heuristic



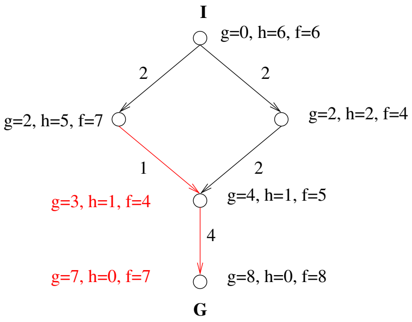
Goal found, but it appears as a child now. Remember that we cannot goal-test a node until it is selected for expansion.

Progress of A* with an inconsistent heuristic



The node with $f = 7$ is selected for expansion. After expansion, the lower node of the diamond gets a new, lower cost.

Progress of A* with an inconsistent heuristic

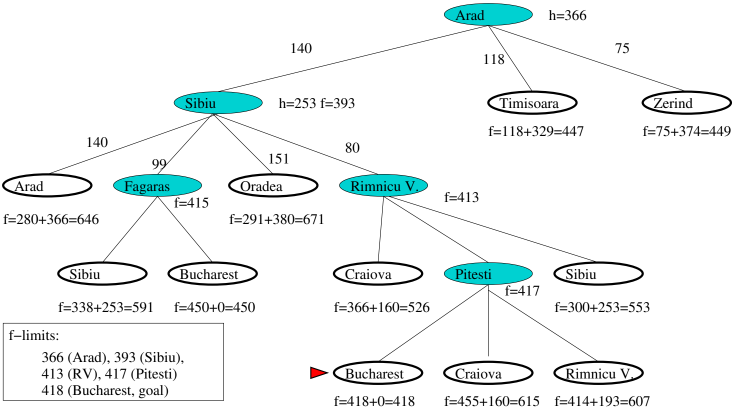


The optimal path to the goal is found.
But nodes had to be reopened.

Iterative deepening A* (IDA*) search

- Idea: perform iterations of DFS. The cutoff is defined based on the f-cost rather than the depth of a node.
- Each iteration expands all nodes inside the contour for the current f-cost, peeping over the contour to find out where the contour lies.

The progress of IDA*



The blue nodes are the ones A* expanded. For IDA*, they define the new f-limit.

IDA* algorithm

function IDA* (*problem*)
returns a solution sequence (or failure)

initialize the *frontier* using the initial state of *problem*
f-limit $\leftarrow$ f-cost(*root*) // f-limit: current f-cost limit
loop do
solution, *f-limit* $\leftarrow$ DFS-CONTOUR(*root*, *f-limit*)
if *solution* is non-null **then return** *solution*
if *f-limit* $= \infty$ **then return** failure

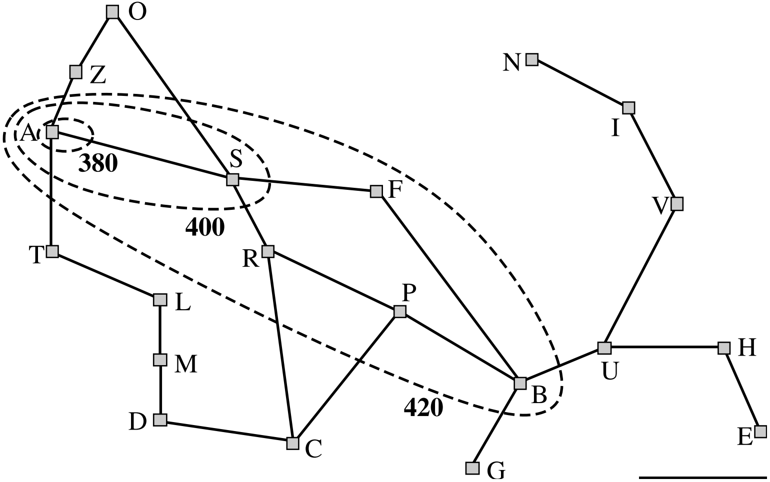
IDA* algorithm (cont'd)

```
function DFS-CONTOUR (node, f-limit)
returns a solution sequence (or failure) and a new f-cost limit

// next-f is initialized to ∞

    if node.f-cost > f-limit then return null, node.f-cost
    if the node contains a goal state then return node, f-limit
    for each child n in node.CHILD-NODES do
        solution, new-f ← DFS-CONTOUR(n, f-limit)
        if solution is not null then return solution, f-limit
        next-f ← MIN(next-f,new-f)
    return null, next-f
```

F-contours for A* search



Properties of IDA*

- *Complete:* Yes, similar to A*.
- *Time:* Depends strongly on the number of different values that the heuristic value can take on.
  8-puzzle: few values, good performance
  TSP: the heuristic value is different for every state. Each contour only includes one more state than the previous contour. If A* expands N nodes, IDA* expands $1 + 2 + \ldots + N = O(N^2)$ nodes.
- *Space:* It is DFS, it only requires space proportional to the longest path it explores. If δ is the smallest operator cost, and f^* is the optimal solution cost, then IDA* will require $b \times f^*/\delta$ nodes to be stored.
- *Optimal:* Yes, similar to A*

Summary

- Consistency enforces the triangle inequality
- If an admissible but not consistent heuristic is used for graph search, we need to adjust path costs when a node is rediscovered
- Heuristic search usually brings dramatic improvement over uninformed search
- Keep in mind that the f-contours might still contain an exponential number of nodes