Chapter 16 Making Simple Decisions

CS5811 - Advanced Artificial Intelligence

Nilufer Onder
Department of Computer Science
Michigan Technological University

Outline

Decision networks

Decision trees

Maximum expected utility (MEU) principle

Preferences

Value of information

Example: Buying tickets

I'm going to buy tickets for two performances at the Rozsa Center. I have two options. I can either buy both of them now at a discount (combined tickets) or I can buy them separately closer to the performance (single tickets). The probability of finding the time for a performance is 0.4. A single ticket costs \$20, and a combined ticket costs \$30. The "value" of going to a performance is \$20. Which ticket should I buy?

The space of outcomes

Probability of finding time ($P(ft_i)$):	0.4
Single ticket:	\$20
Combined ticket:	\$30
Value of going to a performance:	\$20

Option	ft_1, ft_2 (p=0.16)	$ft_1, \neg ft_2$ (p=0.24)	$\neg ft_1, ft_2$ (p=0.24)	$\neg ft_1, \neg ft_2$ (p=0.36)
Combined	cost = \$30 value = \$40 total = \$10	cost = \$30 value = \$20 total = -\$10	cost = \$30 value = \$20 total = -\$10	cost = \$30 value = \$0 total = -\$30
Single	cost = \$40 value = \$40 total = \$0	cost = \$20 value = \$20 total = \$0	cost = \$20 value = \$20 total = \$0	cost = \$0 value = \$0 total = \$0

Computing the expected value

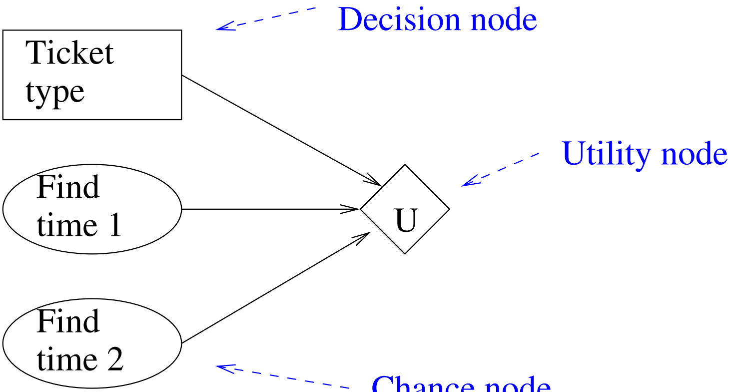
Option	ft, ft (p=0.16)	ft, ¬ft (p=0.24)	¬ ft, ft (p=0.24)	¬ ft, ¬ ft (p=0.36)
Combined	cost = \$30 value = \$40 total = \$10	cost = \$30 value = \$20 total = -\$10	cost = \$30 value = \$20 total = -\$10	cost = \$30 value = \$0 total = -\$30
Single	cost = \$40 value = \$40 total = \$0	cost = \$20 value = \$20 total = \$0	cost = \$20 value = \$20 total = \$0	cost = \$0 value = \$0 total = \$0

The "expected value" of buying a combined ticket is
$0.16 \times 10 + 0.24 \times -10 + 0.24 \times -10 + 0.36 \times -30 = -\$14.0$

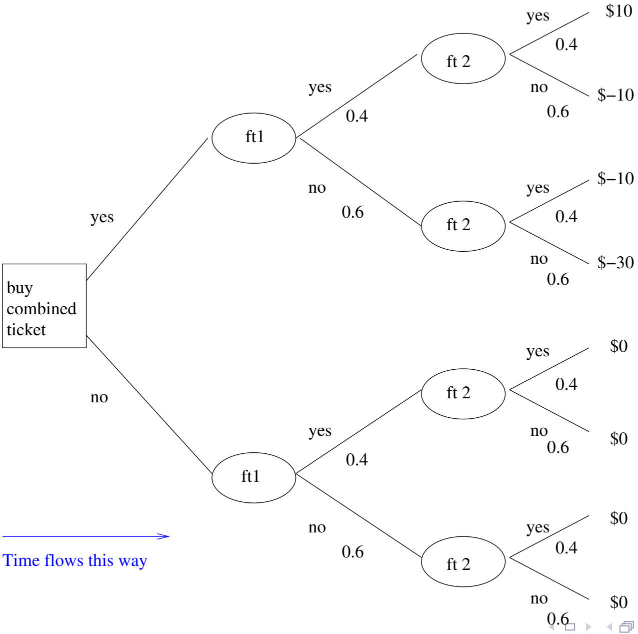
The "expected value" of buying single tickets is \$0.

Therefore, the "rational choice" is to buy single tickets.

The decision network for buying tickets



The decision tree for buying tickets



With a different probability value

- Buying a combined ticket in advance is not a good idea when the probability of attending the performance is low.
- Now, change that probability to 0.9.
- The "expected value" of buying a combined ticket is $0.81 \times 10 + 0.09 \times \text{-}10 + 0.09 \times \text{-}10 + 0.01 \times \text{-}30 = 6.0$
- This time, buying combined tickets is preferable to buying single tickets.

Maximum expected utility (MEU)

Unlike decision making with perfect information, there are now:

- uncertain outcomes
- conflicting goals
- conflicting measure of state quality (not goal/non-goal)

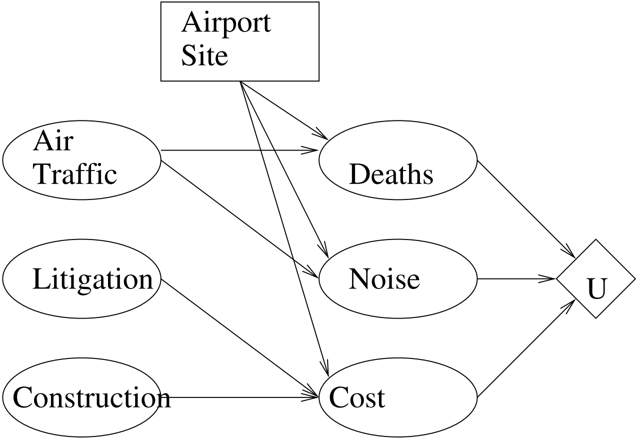
A rational agent should choose the action which maximizes its expected utility (EU), given its knowledge:

$EU(a \mid e) = \sum_s P(\mathrm{RESULT}(a) = s \mid a, e) U(s)$

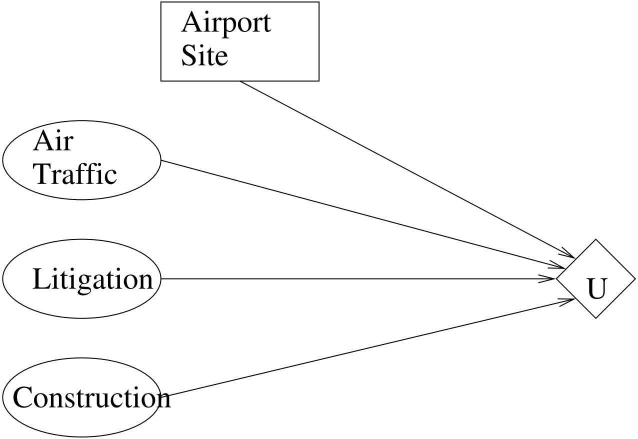
Action $= argmax_a EU(a|e)$[1]

[1] The function $argmax_a$ returns the action a that yields the *max*imum value for the *arg*ument ($EU(a \mid e)$).

Airport siting problem



Simplified decision diagram



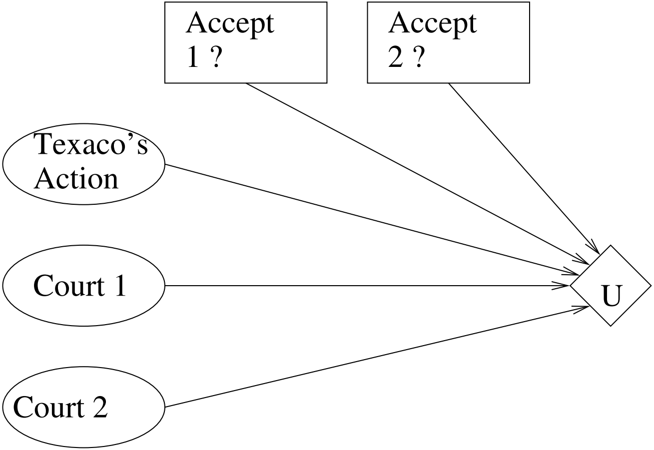
Evaluating decision networks or trees

1. Set the evidence variables for the current state
2. For each possible value of the decision node:
   2.1 Set the decision node to that value.
   2.2 Calculate the posterior probabilities for the parent nodes of the utility node, using a standard probabilistic inference algorithm
   2.3 Calculate the resulting utility for the action
3. Return the action with the highest utility.

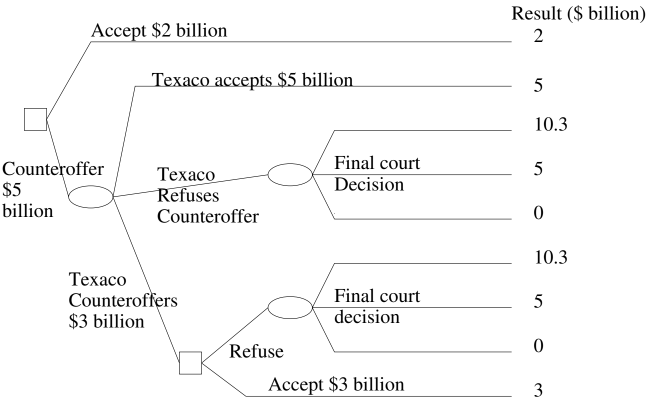
Texaco versus Pennzoil

In early 1984, Pennzoil and Getty Oil agreed to the terms of a merger. But before any formal documents could be signed, Texaco offered Getty Oil a substantially better price, and Gordon Getty, who controlled most of the Getty stock, reneged on the Pennzoil deal and sold to Texaco. Naturally, Pennzoil felt as if it had been dealt with unfairly and filed a lawsuit against Texaco alleging that Texaco had interfered illegally in Pennzoil-Getty negotiations. Pennzoil won the case; in late 1985, it was awarded $11.1 billion, the largest judgment ever in the United States. A Texas appeals court reduced the judgment by $2 billion, but interest and penalties drove the total back up to $10.3 billion. James Kinnear, Texaco's chief executive officer, had said that Texaco would file for bankruptcy if Pennzoil obtained court permission to secure the judgment by filing liens against Texaco's assets. Furthermore Kinnear had promised to fight the case all the way to the U.S. Supreme Court if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty. In April 1987, just before Pennzoil began to file the liens, Texaco offered Pennzoil $2 billion to settle the entire case. Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that a settlement of between $3 billion and $5 billion would be fair.

Liedtke's decision network



Liedtke's decision tree



Issues

- How does one represent preferences?
- How does one assign preferences?
- Where do we get the probabilities from?
- How to automate the decision making process?

Preferences

- An agent must have preferences among:
  - Prizes: A, B
  - Lotteries: list of outcomes with associated probabilities
    $L = [p_1, s_1; p_2, s_2; \ldots p_n, s_n]$
- Notation:
  - A $\succ$ B: A is preferred to B
  - A $\sim$ B: indifference between A and B
  - A $\succeq$ B: B not preferred to A

Axioms of utility theory

- Orderability
- Transitivity
- Continuity
- Subsitutability
- Monotonicity
- Decomposibility

Orderability and Transitivity

Orderability: The agent cannot avoid deciding:

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity: If an agent prefers A to B and prefers B to C, then the agent must prefer A to C.

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity and Substitutability

Continuity: If some state B is between A and C in preference, then there is some probability p such that

$$A \succ B \succ C \Rightarrow \exists p\,[p, A; 1 - p, C] \sim B$$

Substitutability: If an agent is indifferent between two lotteries A and B, then the agent is indifferent between two more complex lotteries that are the same except that B is substituted for A in one of them.

$$(A \sim B) \Rightarrow [p, A; 1 - p, C] \sim [p, B; 1 - p, C]$$

Monotonicity and Decomposability

Monotonicity: If an agent prefers A to B, then the agent must prefer the lottery that has a higher probability for A.

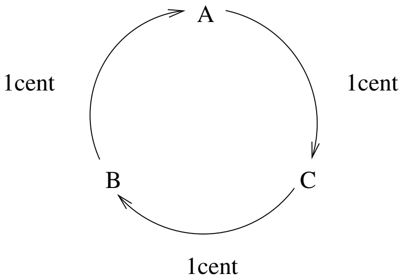
$$A \succ B \Rightarrow (p \geq q) \Leftrightarrow [p, A; (1-p), B] \succeq [q, A; (1-q), B]$$

Decomposability: Two consecutive lotteries can be compressed into a single equivalent lottery

$$[p, A; (1-p), [q, B; (1-q), C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$$

Rational preferences

The axioms are constraints that make preferences rational.
An agent that violates an axiom can exhibit irrational behavior
For example, an agent with intransitive preferences can be induced to give away all of its money.
If $X \succ Y$, give 1 cent to trade Y for X:



Utility Theory

- Theorem: (Ramsey, 1931, von Neumann and Morgenstern, 1944):
  Given preferences satisfying the constraints
  there exists a real-valued function U such that
  $U(A) \geq U(B) \Leftrightarrow A \succeq B$
  $U(A) = U(B) \Leftrightarrow A \sim B$
  $U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$
- The first type of parameter represents the deterministic case
- The second type of parameter represents the nondeterministic case, a *lottery*

Utility functions

- A *utility function* maps states to numbers: $U(S)$
- It expresses the desirability of a state (totally subjective)
- There are techniques to assess human utilities
- utility scales
  - normalized utilities: between 0.0 and 1.0
  - micromorts: one-millionth chance of death
    useful for Russian roulette, paying to reduce product risks etc.
  - QALYs: quality-adjusted life years
    useful for medical decisions involving substantial risk

Money

- Money does not usually behave as a utility function
- Empirical data suggests that the value of money is logarithmic
  - For most people getting \$5 million is good, but getting \$6 million is not 20% better
  - Textbook's example: get \$1M or flip a coin for \$3M?
  - For most people getting in debt is not desirable but once one is in debt, increasing that amount to eliminate debts might be desirable

Value of information

- An oil company is hoping to buy one of n distinguishable blocks of ocean drilling rights
- Exactly one of the blocks contains oil worth C dollars
- The price of each block is C/n dollars
- If the company is risk-neutral, then it will be indifferent between buying a block and not buying one

Value of information (cont'd)

- n blocks, C worth of oil in one block, each block C/n dollars
- A seismologist offers the company the results of a survey of block number 3, which indicates definitely whether the block contains oil.
- How much should the company be willing to pay for the information?

Value of information (cont'd)

- n blocks, C worth of oil in one block, each block C/n dollars. Value of information about block number 3?
- With probability $1/n$ the survey will indicate oil in block 3. In this case, the company will buy block 3 for C/n dollars and make a profit of $C - C/n = (n-1)C/n$ dollars
- With probability $(n-1)/n$, the survey will show that the block contains no oil, in which case the company will buy a different block. Now the probability of finding oil in one of the blocks changes from $1/n$ to $1/(n-1)$ so the company makes an expected profit of $C/(n-1) - C/n = C/(n(n-1))$ dollars.

Value of information (cont'd)

- n blocks, C worth of oil in one block, each block C/n dollars. Value of information about block number 3?
- The expected profit given the survey information is $\frac{1}{n} \times \frac{(n-1)C}{n} + \frac{n-1}{n} \times \frac{C}{n(n-1)} = C/n$
- The information is worth as much as the block itself!

Issues revisited

- How does one represent preferences?
  (a numerical utility function)
- How does one assign preferences?
  (compute $U(Result_i(A))$—requires search or planning)
- Where do we get the probabilities from?
  (compute $U(Result_i(A)|Do(A), E)$—requires a complete causal model of the world and NP-hard inference)
- How to automate the decision making process?
  (influence diagrams)

Summary

- Can reason both qualitatively and numerically with preferences and value of information
- When several decisions need to be made, or several pieces of evidence need to be collected it becomes a *sequential decision problem*
  - value of information is nonadditive
  - decisions/evidence are order dependent

Sources for the slides

- AIMA textbook (3^{rd} edition)
- AIMA slides (http://aima.cs.berkeley.edu/)
- Clemen, Robert T. Making Hard Decisions: An Introduction to Decision Analysis. Duxbury Press, Belmont, California, 1990.