CE3501 ENVIRONMENTAL ENGINEERING FUNDAMENTALS
ENVIRONMENTAL PHYSICS PROBLEM SET 1
FALL 2005 SOLUTIONS

Problem numbers are those in the text. Note that the first two problems will not be covered in lecture. If you cannot get the answer or do not understand the method, then ask for help.

1. Problem 2-18 NO SOLUTION PROVIDED
2. Problem 2-21 NO SOLUTION PROVIDED
3. Problem 4-2, but replace the example problems given in your text with the following:
   a) The Montreal Protocol banned the production of most types of CFCs (chlorofluorocarbons), in order to protect the stratospheric O_3 layer. If there are no longer any CVC emissions to the atmosphere, and if CFCs continue to be destroyed with a first-order rate constant of k (units of 1/yr), how long will it take the CFC concentration to drop to 50% of the current value?

i) What control volume should be used? The earth's atmosphere.
ii) Are conditions changing with time? Yes.
iii) Is the problem steady state or non-steady state? Non-steady state. Concentrations are declining with time due to reactions exceeding emissions.
iv) If the chemical reacting in the control volume? Yes, CFCs are being destroyed.
v) Is the chemical conservative? No, it is nonconservative due to the chemical destruction.

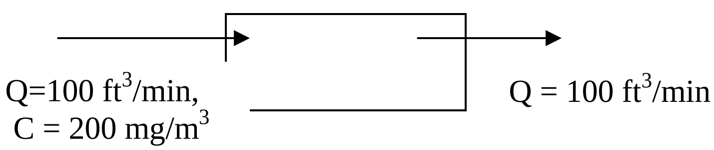
   b) If mercury (Hg) is released into the waters of Lake Superior from underwater mine tailings at a rate of A kg/yr, and if Hg is removed from the water column by continuous biological and chemical processes with a rate constant of k (1/month), what is the resulting concentration of mercury in the lake?

i) What control volume should be used? The water column of Lake Superior.
ii) Are conditions changing with time? No.
iii) Is the problem steady state or non-steady state? Steady state. Sources equal sinks, and no change in rates has occurred recently.
iv) If the chemical reacting in the control volume? Yes, Hg is being produced and consumed.
v) Is the chemical conservative? No, it is nonconservative due to the chemical and biological removal processes.

   c) Carbon monoxide (CO, a chemically stable but toxic gas) enters a home through a small leak in a chimney leading from a fireplace, in which a small fire smolders continuously all night. Clean air enters the house(through leaks and gaps in windows and doors) at a rate Q, and an equal amount of air leaves the house. What is the resulting concentration of CO in the house?

i) What control volume should be used? The house.
ii) Are conditions changing with time? No.
iii) Is the problem steady state or non-steady state? Steady state. By the end of the night, a steady state may have been reached in which sources (leak from fire) equal outputs (flushing). If the fire goes out or changes its emission rate, non-steady state conditions would exist.
iv) Is the chemical reacting in the control volume? No.
v) Is the chemical conservative? Yes.

4. Problem 4-5



This is a non-steady state problem with a conservative substance. We are asked to find the time required to reach a concentration of 100 mg/m^3 in a room of 1000 ft^3. The mass balance equation is:

$$V\frac{dC}{dt} = \dot{m}_{in} - \dot{m}_{out} = QC_{in} - QC$$

The integrated solution to this equation is:

$$C = C_0 e^{-\frac{Q}{V}t} + C_{in}\left(1 - e^{-\frac{Q}{V}t}\right)$$

The initial poison concentration in the room, C_0, was zero, and to goal is to find the time until C = 100 mg/m^3.

$$t = -\frac{V}{Q}\ln\left(\frac{C_{in} - C}{C_{in}}\right)$$

$$t = \frac{-1000\,ft^3}{100\,\frac{ft^3}{\min}}\ln\left(\frac{200-100}{200}\right) = 6.9\min$$

You can relax, you've got nearly 7 minutes.

5. Problem 4-6

a) What is the mass flux of CO into the ice rink interior?

The mass flux will be the sum of the intake in the ventilation air plus the emissions from the Zamboni.

$$\dot{m}_{in} = Q_1C_1 + E = \left(3\frac{m^3}{s}\right)\left(10\frac{mg}{m^3}\right) + 8\frac{mg}{s} = 38\frac{mg}{s}$$

b) Find the steady state concentration of CO within the ice rink.
At steady state, the inputs must equal the outputs for this conservative substance.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$38\frac{mg}{s} = Q \cdot C$$

$$C = \frac{38\frac{mg}{s}}{3\frac{m^3}{s}} = 12.7\frac{mg}{m^3}$$

6. Problem 4-8

a) What is the BOD concentration just downstream of the discharge point?

$$C_0 = \frac{C_d Q_d}{Q_{u/s} + Q_d}$$

$$C_0 = \frac{0.9\frac{m^3}{s} \cdot 50\frac{mg}{L}}{8.7\frac{m^3}{s} + 0.9\frac{m^3}{s}} = 4.69\frac{mg}{L}$$

b) If the stream has a cross-sectional area of $10m^2$, what would the BOD concentration be 50 km downstream? The decay constant is $0.2\ d^{-1}$.

$C = C_o e^{-kx/u}$

The velocity in the stream is Q/A or $(9.6m^3/s)/10m^2 = 0.96$ m/s

Therefore the concentration may be calculated as:

$$C = C_0 \exp\left(\frac{-kx}{u}\right) = 4.69 \exp\left(\frac{-0.2d^{-1} 50x10^3 m}{0.96\frac{m}{s} \cdot 24 \cdot 3600\frac{s}{d}}\right) = 4.16\frac{mg}{L}$$

7. Problem 4-11 parts (a) and (b)

a) Calculate the steady-state concentration of pollutant in the lake. You are given:

$V = 20x10^3\ m^3$

$Q_s = 0.5\ m^3/s$, $C_s = 100$ mg/L

$k = 0.2\ d^{-1}$, $Q = 5\ m^3/s$

At steady state, the mass balance equation reduces to:

$$\dot{m}_{in} = \dot{m}_{out} + \dot{m}_{rxn}$$

$$Q_s C_s = QC + kCV$$

Rearranging the equation to solve for C yields:

$$C = \frac{Q_s C_s}{Q + kV} = \frac{0.5\frac{m^3}{s} \cdot 100\frac{mg}{L}}{5\frac{m^3}{s} + 0.2d^{-1}\left(\frac{d}{24 \cdot 3600s}\right) \cdot 20x10^3 m^3} = 9.9\frac{mg}{L}$$

b) What is the retention time for water in this lake?

The water retention time is simply V/Q or $20\times10^3 m^3/5\ m^3/s = 4000$ s or 1.11 hr

8. Problem 4-16

Derive the expressions for V and C_o/C_{in} given in Table 4-2.

For a CMFR at SS:

$$\dot{m}_{in} = \dot{m}_{out} + \dot{m}_{rxn}$$

$$QC_{in} = C\left(Q + kV\right)$$

This equation may be rearranged to solve for either C/C_{in} or V to yield:

$$V = \frac{Q}{k}\cdot\left(\frac{C_{in}}{C} - 1\right)$$

$$C = \frac{QC_{in}}{Q + kV} = \frac{C_{in}}{1 + \frac{kV}{Q}}$$

For a PFR at SS:

$C = C_o e^{-kV/Q}$

This may be rearranged to solve for V

$V = -Q/k \ln(C/C_{in})$

9. The temperature of well-water entering a hot water heater tank (size of 40 gal) is 5^oC. A hot water tap has been left on at a flow of 1 gal/min, but power to the heater has been off, so the water temperature in the tank is 5^oC. If the heater is now turned on (power = 5 kW), while the tap is left on,
   a) What temperature will eventually be reached if the water and heater are left on indefinitely?
   b) How long will it take to reach a temperature of 20^oC?

a) indicate the control volume;

The control volume is the inside of the insulated hot water tank

b) Indicate each of the energy flows into and out of the control volume;

Energy inputs include the inflowing water and the 5 kW from the heater

Energy outputs include only the water leaving the tank

c) Indicate whether the problem is steady-state or non-steady-state.

The problem for part a is SS: we are asked to find the temperature eventually reached once conditions stop changing. For part b the problem is NSS; turning the heater on represents a change in conditions.

a)

$$\dot{E}_{in} = \dot{E}_{out}$$

$$QT_{in}c_w + P_{heater} = QTc_w$$

$$T = \frac{QT_{in}c_w + P_{heater}}{Qc_w} = \frac{1\frac{gal}{\min}\cdot 3.78\frac{L}{gal}\cdot 10^3\frac{g}{L}\cdot\frac{1\min}{60\sec}\cdot 5^o C\cdot 1\frac{cal}{g\cdot^0 C}\cdot 4.184\frac{J}{cal} + 5kW\cdot 10^3\frac{W}{kW}\cdot 1\frac{J/s}{W}}{1\frac{gal}{\min}\cdot 3.78\frac{L}{gal}\cdot 10^3\frac{g}{L}\cdot\frac{1\min}{60\sec}\cdot 1\frac{cal}{g\cdot^0 C}\cdot 4.184\frac{J}{cal}}$$

$$T = 23.9^o C$$

b) This is a NSS problem; the mass balance equation must be integrated and solved for time when $T_{out} = 20^oC$. First, it is useful to calculate Q/V = (1 gal/min)/40gal = 0.025 $\min^{-1}$.

$$\frac{dE}{dt} = \dot{E}_{in} + \dot{E}_{htr} - \dot{E}_{out} = QT_{in}c_w + P_{htr} - QTc_w = Vc_w\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{Q}{V}T_{in} + \frac{P_{htr}}{Vc_w} - \frac{Q}{V}T$$

$$\int\frac{-\frac{Q}{V}dT}{\left(\frac{Q}{V}T_{in} + \frac{P}{Vc_w} - \frac{Q}{V}T\right)} = \int -\frac{Q}{V}dt$$

$$\ln\left[\frac{\frac{Q}{V}T_{in} + \frac{P}{Vc_w} - \frac{Q}{V}T}{\frac{Q}{V}T_{in} + \frac{P}{Vc_w} - \frac{Q}{V}T_0}\right] = -\frac{Q}{V}t$$

$$t = -\frac{1}{0.025\min^{-1}}\cdot$$

$$\ln\left[\frac{0.025\min^{-1}\cdot 5^o C\cdot\frac{1\min}{60s} + \frac{5kW\cdot 10^3\frac{W}{kW}\cdot 1\frac{J/s}{W}}{40gal\cdot 3.78\frac{L}{gal}\cdot 10^3\frac{g}{L}\cdot 1\frac{cal}{g\cdot^o C}\cdot 4.184\frac{J}{cal}} - 0.025\min^{-1}\cdot 20^o C\cdot\frac{1\min}{60s}}{\frac{5kW\cdot 10^3\frac{W}{kW}\cdot 1\frac{J/s}{W}}{40gal\cdot 3.78\frac{L}{gal}\cdot 10^3\frac{g}{L}\cdot 1\frac{cal}{g\cdot^o C}\cdot 4.184\frac{J}{cal}}}\right]$$

$$t = 62.6\min$$