Name ________________________

CE3501 Fundamental of Environmental Engineering
Environmental Physics – Hour Exam
Fall 2005
SOLUTIONS

Short answer (4 pts each). Circle all correct answers. A page of equations is given at the end of the exam.

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1. The equation relating diffusive flux density to the concentration gradient is called
    a. Newton's Law
    b. Stoke's Law
    c. Fick's Law
    d. Darcy's Law
    e. None of the above

2. The equation relating water flow rate through a porous medium to the hydraulic gradient is called
    a. Newton's Law
    b. Stoke's Law
    c. Fick's Law
    d. Darcy's Law
    e. None of the above

3. What three (3) forces are included in the force balance for calculation of the terminal settling velocity (i.e., gravitational settling):
    a. Gravitational force, Drag force, Centrifugal force
    b. Gravitational force, Drag force, Buoyancy force
    c. Gravitational force, Electrostatic force, Buoyancy force
    d. Gravitational force, Centrifugal force, Drag force
    e. None of the above

4. Which of the following statements are TRUE?
    a. Turbulent Dispersion depends on the molecular properties of the compound;
    b. The rate of molecular diffusion depends on the velocity of fluid flow;
    c. Mechanical dispersion occurs primarily in pipes;
    d. All three types of dispersion are affected by temperature because of its effect on fluid viscosity;
    e. None of the above;

5. Which of the following statements are TRUE?
    a. A PFR is more efficient than a CSTR of the same volume;
    b. A PFR is more susceptible to spikes in influent concentrations than is a CSTR:
    c. For a substance undergoing a first-order decomposition reaction, the average concentration in a PFR is higher than in a CSTR of the same volume;

d. The acronyms CSTR and CMFR are interchangeable;
e. None of the above.

6. A lake with a hydraulic retention time of 150 days is fed by a tributary stream with a flow of $7.5\text{x}10^5$ m^3/day. A chemical that decays according to first order kinetics (k = 0.05 d^{-1}) is present in the stream at a concentration of 100 mg/L. What is the steady state chemical concentration in the lake? [10 pts]

SOLUTION:

$$V\frac{dC}{dt} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{reaction} = 0$$

$$QC_{in} = QC + kCV$$

$$C = \frac{QC_{in}}{Q + kV} = \frac{\frac{Q}{V}C_{in}}{\frac{Q}{V} + k} = \frac{\frac{C_{in}}{\Theta}}{\frac{1}{\Theta} + k}$$

$$C = \frac{\frac{100mg \cdot L^{-1}}{150d}}{\frac{1}{150d} + 0.05d^{-1}} = 11.8mg \cdot L^{-1}$$

7. Chloride concentrations in Lake Superior are increasing at a rate of 1%/year because of Cl^- inputs from road salt and from the discharges from a wastewater treatment plants. Given that the water residence time is 170 years, how long would it take for the lake to return to within 95% of the pre-European-influence concentration if all human inputs of Cl^- were stopped suddenly. (15 pts)

SOLUTION

$$V\frac{dC}{dt} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{reaction} = -\dot{m}_{out}$$

$$\frac{dC}{dt} = \frac{-Q}{V}C = \frac{-C}{\Theta}$$

$$\int \frac{dC}{C} = \int \frac{-1}{\Theta} dt$$

$$C = C_o e^{\frac{-t}{\Theta}}$$

$$t = -\Theta \ln\left(\frac{C}{C_o}\right) = -170yr \cdot \ln\left(\frac{0.05C_o}{C_o}\right) = 509yr$$

If the problem asks when does it decline by 5%, then the answer would be 8.7 yr.

8. A river is flowing with a velocity of 20 km•d^{-1}. A chemical which is known to decay according to first order kinetics (k = 0.1 d^{-1}) is found in this river 50 km downstream of an industrial discharge at a concentration of 100 mg•L^{-1}. The industry discharges at a flow of 1 m^3•s^{-1}. The substance is not present upstream of the discharge, where the flow is 9 m^3•s^{-1}. What is the concentration of the chemical in the undiluted waste? (20 pts)

SOLUTION
The first step is to use the PFR equation to find the concentration at the pont of discharge. [10 pts]
$C = C_o \exp(-kx/u)$
$C_o = C/\exp(-kx/u) = (100\ \text{mg/L})/\exp[(-0.1\text{d}^{-1})(50\text{km})/(20\text{km/d})] = 128\ \text{mg/L}$

The next step is to use the mixing equation to find the concentration in the waste discharge [10 pts]

$$C_o Q_o = C_{in} Q_{in} + C_w Q_w$$

$$C_w = \left(C_o Q_o - C_{in} Q_{in}\right) \cdot \frac{1}{Q_w}$$

$$C_w = \left(128\frac{mg}{L} \cdot 10\frac{m^3}{s} - 0\frac{mg}{L} \cdot 9\frac{m^3}{s}\right) \cdot \frac{1}{1\frac{m^3}{s}} = 1280\frac{mg}{L}$$

Diffusion/dispersion
9. Torch Lake was the dumping ground for 200 million tons of copper mining wastes. These wastes in the sediments continue to release copper into the porewater from whence it diffuses upwards into the lake. The U.S.EPA deemed it too expensive to treat the sediments of the lake and is relying instead on natural processes to purify the lake. The two natural processes are flushing of the lake with clean river water and burial of the contaminated sediments under new sediments. You are to determine if this is a feasible plan by comparing the rate of diffusion of copper out of the sediments when the mining ended (1968) with the rate 40 years later when the contaminated sediments have been buried under 6 cm of "uncontaminated" sediments. Assume that the copper concentration in the porewaters of the contaminated sediments remains at 800 mg/m^3 and that the concentration in the lake remains at 30 mg/m^3. The effective diffusion coefficient for copper is 0.132 m^2/yr. **Calculate the diffusive fluxes out of the sediment immediately after mining when only 0.3 cm of sediments cap the mine wastes and 40 years later when 6 cm of sediments cap the mine wastes. Is the "No Action" plan a reasonable choice based on your calculation? [15 pts]**

SOLUTION

The diffusive flux is calculated with Fick's Law:

$$J = -D\frac{dC}{dx}$$

$$J_{1968} = -0.132\frac{m^2}{yr}\cdot\frac{\left(800-30\dfrac{mg}{m^3}\right)}{0.003m} = 33,880\frac{mg}{m^2yr}$$

$$J_{2008} = -0.132\frac{m^2}{yr}\cdot\frac{\left(800-30\dfrac{mg}{m^3}\right)}{0.06m} = 1,694\frac{mg}{m^2yr}$$

$$\frac{J_{2008}}{J_{1968}} = \frac{1,694\dfrac{mg}{m^2yr}}{33,880\dfrac{mg}{m^2yr}} = 5\%$$

After 40 years of natural sedimentation, the diffusive input of copper from the sediments should be only 5% of what it was immediately after mining ceased. From this standpoint, the "No Action" option seems a reasonable choice.

10. A stormwater detention pond has a mean depth of 1 m and a surface area of 50,000 m^2. Following a storm, the inflow rate is 1 m^3/s. What is the minimum size of particle that the pond will effectively remove under these conditions? Particle density is 2.5 g/cm^3, water viscosity is 0.01185 g/cm-s, and water density is 1 g/cm^3. (20 pts)

SOLUTION:

The first step is to find the minimum settling velocity removed by the pond.

$V_{min} = h/\theta = Q/A = (1\ m^3/s)/50,000\ m^2 = 2x10^{-5} m/s = 1.73\ m/d$

The second step is to calculate the size of particle that has this settling velocity.

$$v_s = \frac{g\left(\rho_{particle} - \rho_{fluid}\right)D_p^2}{18\mu}$$

$$D_p = \sqrt{\frac{v_s \cdot 18\mu}{g\left(\rho_{particle} - \rho_{fluid}\right)}}$$

$$D_p = \sqrt{\frac{2x10^{-5}\dfrac{m}{s}\cdot 18\cdot 0.01185\dfrac{g}{cm\cdot s}\cdot 100\dfrac{cm}{m}}{980\dfrac{cm}{s^2}\left(2.5-1\dfrac{g}{cm^3}\right)}}$$

$$D_p = 5.4x10^{-4}\,cm = 5.4\mu m$$

EQUATIONS

$$v_d = \frac{Q}{A} = -K \cdot \frac{dh}{dx}$$

$$v_s = \frac{D^2 \cdot g \cdot \left(\rho_p - \rho_l\right)}{18 \cdot \mu}$$

$$J = -D \cdot \frac{dC}{dx}$$

$$\Theta = \frac{V}{Q}$$

$$C_{mb} = \frac{C_{up} \cdot Q_{up} + C_{in} \cdot Q_{in}}{Q_{up} + Q_{in}}$$