ENG5300 Engineering Applications in the Earth Sciences:

River Velocity

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I. Introduction

It is important to be able to calculate river velocity for almost anything having to do with structures that reside in it (docks), span it (bridges) or must control its flow (culverts, levees, and bend deflectors). The complexity of the geometry and morphology (cross-section shape, bottom type) of most rivers does not allow for a theoretical analysis of rivers. Thus, empirical relationships are often used. The two most common methods are the Manning and Chézy formulas:

Manning Formula: $$V = \frac{u_m}{n} \cdot R_h^{2/3} \cdot S^{1/2} \tag{1}$$

Chézy Formula: $$V = u_c \cdot C \cdot R_h^{1/2} S^{1/2} \tag{2}$$

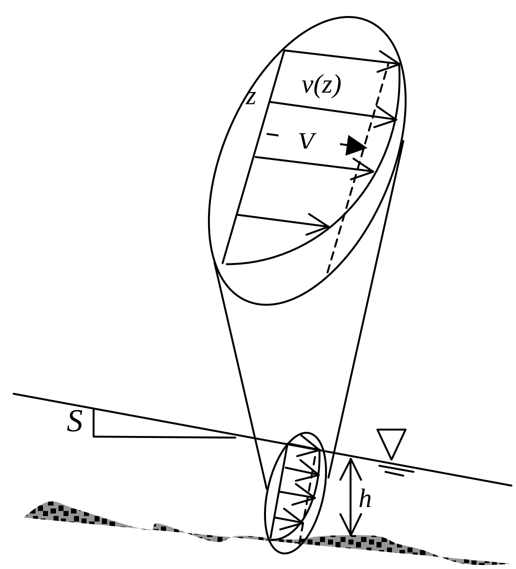
Where V is the *average stream velocity*, u is a unit conversion factor for the Manning (subscript m) and Chézy (subscript c) formulas (see Table 1 for the units for Equations 1 and 2), R_h is the hydraulic radius (see Box 2), S is the slope of the free-water surface, n is the "Manning's n," and C is the "Chézy's C," both of which are lumped parameters reflecting the stream morphology (see Table 2).

Equations 1 and 2 are very similar in form, the only differences are the unit conversion factors, lumped coefficients, n and C, and the power to which the hydraulic radius is raised.

The Manning and Chézy coefficents are empirically based. Reference books on hydraulics and hydrology contain tabulations of n and, sometimes, C, for many types of streams and other hydraulic structures, such as culverts, pipes, canals (see Table 2).

Box 1: *Average Stream Velocity*

Stream velocity (V) is primarily a function of the stream area (A), morphology, and slope (S). The average stream velocity reflects the velocity which when multiplied by the cross-sectional area of the stream (area = width·depth: $A = w \cdot h$) gives the stream discharge (Q).



Box 2: *Hydraulic Radius*

The hydraulic radius (R_h) is sort of like an "equivalent radius" that represents a combined effect of cross-sectional area (A) of bottom/bank shape (wetted perimeter, P): $R_h = A/P$. For wide, shallow streams (depth < 0.05 width), the hydraulic radius is essentially the average depth, $R_h \approx h$.

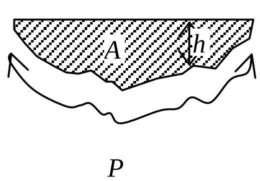


Table 1. Unit conversion factors for Manning and Chézy Formulas.

Units on *V*	**Units on R_h**	u_m	u_c
feet/second	feet	1.49	1.00
meters/second	meters	1.00	0.552
cm/s	cm	4.64	5.52

Table 2. Some typical values of Manning's *n* for various types of streams and aqueducts.

Channel Condition(s)	***n***	**Variability**	**Roughness (mm)**
Glass	0.010	±0.002	0.3
Painted Steel	0.014	±0.003	1
Unfinished Concrete	0.014	±0.002	1.0
Corrugated Metal	0.016	±0.005	37
Masonry Rubble	0.025	±0.005	80
ravelly	0.025	±0.005	80
Natural Clean & Straight	0.030	±0.005	240
Major Rivers	0.035	±0.010	500
Sluggish Reaches & Deep Weedy Pools	0.065	±0.015	900

Notes: (1) Chézy *C* can be equated to the Manning *n* and the hydraulic radius (or depth) according to: $C = \frac{u_m R_h^{1/6}}{u_c n}$, which can be derived by equating the Chézy and Manning Formulas.
(2) In the absence of a published value, the value of *n* can be estimated based on the roughness: $n = 0.121\ \varepsilon^{1/6}$ where the roughness (ε) is in millimeters.
(3) The values given above were taken from lists that cite V. T. Chow's book, *Open Channel Hydraulics* (McGraw-Hill, New York, 1959) as the most complete reference on these formulas.

II. Laboratory Activity

A demonstration stream bed (flume) can be fabricated from PVC pipe (split in half lengthways) or rain gutters to conduct experiments for validating the Manning and/or Chézy Formulas or for calculating Manning *n* and/or Chézy *C* values. Granular materials, such as sands and gravels of different sizes, can be glued (use a waterproof epoxy) to the flume to create a variety of rough surfaces. The flow in the flume and/or its slope can be varied and the depth of flow measured (the flow and slopes must also has be measured). The Manning *n*, for example, can then be calculated using the example data form provided on the next page.

Flume Data Sheet

Measured Property:	**Volume of Flow**	**Fill Time**	**Flow Rate**	**Width**	**Average Depth**	**Cross-Sectional Area**	**Average Velocity**	**Up-stream Height**	**Down-stream Height**	**Hori-zontal Length**	**Slope**	**Manning *n***
Abbreviation:	*Vol*	t	Q	w	h	A	V	E_u	E_d	L	S	n
Obtained by:	Graduated Cylinder	Stopwatch	*Vol/t*	Ruler	Ruler	$w \cdot h$	Q/A	Ruler	Ruler	Ruler	$\frac{E_u - E_d}{L}$	$\frac{u_m h^{2/3} S^{1/2}}{V}$
Units:												
Material Type	**Measured Data**											
Made-up example	241 cm^3	24 s	10 cm^3/s	4.0 cm	0.10 cm	0.4 cm^2	25 cm/s	11 cm	0 cm	70 cm	0.16	0.040

III. Applications

Either using experimentally determined Manning *n* numbers or published values, a variety of channel-area-slope problems can be made up. Below are two examples:

1. **Culvert Design**

   A culvert is needed beneath a road to accommodate the spring peak flow. The slope of the culvert is 5% and it is flowing at its most efficient depth, where the depth of flow is 82% full. The maximum discharge velocity such that there is no scour (erosion) on the downstream end is 10 ft/s. Determine the size of culvert needed. To do this problem, one would also need to know that the hydraulic radius of the most efficient depth of flow (maximum velocity) is 22% greater than the hydraulic radius flowing full ($R_{h,full}$ = diameter/4): $R_{h,max} = 1.22\ R_{h,full}$

   Steps:

   a. Select appropriate Manning *n* from Table 2: $n = 0.022$

   b. Determine minimum hydraulic radius, R_h, using Manning Formula so that discharge velocity is at 10 ft/s:

   $$R_{h,max} = (n\ V / S^{1/2})^{3/2} = (0.022\ 10 / 0.05^{1/2})^{3/2} = 0.98 \text{ ft}$$

   c. Using the added information about the relationship between the most efficient hydraulic radius and $R_{h,full}$ determine the diameter:

   Diameter = $4R_{h,full} = 4R_{h,max}/1.22 = 4 \cdot 0.98/1.22 = 3.20$ ft = 38.4 inches (culverts come in nominal diameters to the nearest 2”, so select the next bigger size: 40”).

2. **Estimate Stream Velocity**

   Estimate the discharge of a stream at a section that is 26 feet wide with an average depth of 6”. The stream bottom is a mixture of sand and gravel.

   Steps:

   a. Assume $R_h = h = 6/12 = 0.5$ ft

   b. Estimate the river slope using a topographic map:

   S = contour interval/distance between contours at stream section

   S = 33 ft / 5 miles = 33 ft / 26,000 ft = 0.0012

   c. Estimate appropriate *n* from Table 2, choose 0.033 (not-so clean, not-so straight)

   d. Calculate *V* using Manning Formula, choose appropriate unit conversion (1.49 for ft and seconds):

   $V = 1.49\ 0.5^{2/3}\ 0.0012^{1/2} / 0.033 = 1.0$ ft/s

   e. Calculate flow using the average stream velocity and cross-sectional area:

   $Q = V \cdot A = 1.0 \text{ ft/s} \cdot 26 \text{ ft} \cdot 0.5 \text{ ft} = 13 \text{ cfs}$

IV. Appendix: Tabulations[i] of Manning *n*.

Dingman, S.L., *Physical Hydrology*, 2nd ed., Prentice Hall, Upper Saddle River, NJ, 2002.

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TABLE 9-6
Values of Manning's *n* for Channels of Various Types[a]

Type of Channel and Description	*n* Minimum	*n* Normal	*n* Maximum
Minor streams (top width at flood stage <100 ft)			
Streams on plain			
1. Clean, straight, full stage, no riffles or deep pools	0.025	0.030	0.033
2. Same as above, but more stones and weeds	0.030	0.035	0.040
3. Clean, winding, some pools and shoals	0.033	0.040	0.045
4. Same as above, but some weeds and stones	0.035	0.045	0.050
5. Same as above, but lower stages, more ineffective slopes and sections	0.040	0.048	0.055
6. Same as 4, but more stones	0.045	0.050	0.060
7. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
8. Very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150
Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
1. Bottom: gravels, cobbles, and few boulders	0.030	0.040	0.050
2. Bottom: cobbles with large boulders	0.040	0.050	0.070
Floodplains			
Pasture, no brush			
1. Short grass	0.025	0.030	0.035
2. High grass	0.030	0.035	0.050
Cultivated areas			
1. No crop	0.020	0.030	0.040
2. Mature row crops	0.025	0.035	0.045
3. Mature field crops	0.030	0.040	0.050
Brush			
1. Scattered brush, heavy weeds	0.035	0.050	0.070
2. Light brush and trees, in winter	0.035	0.050	0.060
3. Light brush and trees, in summer	0.040	0.060	0.080
4. Medium to dense brush, in winter	0.045	0.070	0.110
5. Medium to dense brush, in summer	0.070	0.100	0.160
Trees			
1. Dense willows, summer, straight	0.110	0.150	0.200
2. Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
3. Same as above, but with heavy growth of sprouts	0.050	0.060	0.080
4. Heavy stand of timber, a few down trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
5. Same as above, but with flood stage reaching branches	0.100	0.120	0.160
Major streams (top width at flood stage >100 ft)[b]			
Regular section with no boulders or brush	0.025		0.060
Irregular and rough section	0.035		0.100

[a] The values of u_x and u_m for the common unit systems are

u_x = SI: 0.551; cgs: 5.52; English: 1.00

u_m = SI: 1.00; cgs: 4.64; English: 1.49

[b] The *n* value is less than that for minor streams of similar description, because banks offer less effective resistance.

Data from Chow (1959).

White, F.M., *Fluid Mechanics*, McGraw-Hill, New York, NY, 1979.

Table 10.1
EXPERIMENTAL VALUES OF MANNING'S *n* FACTOR†

	n	Average roughness height, ε: ft	Average roughness height, ε: mm
Artificial lined channels:			
Glass	0.010 ± 0.002	0.0011	0.3
Brass	0.011 ± 0.002	0.0019	0.6
Steel, smooth	0.012 ± 0.002	0.0032	1.0
Painted	0.014 ± 0.003	0.0080	2.4
Riveted	0.015 ± 0.002	0.012	3.7
Cast iron	0.013 ± 0.003	0.0051	1.6
Cement, finished	0.012 ± 0.002	0.0032	1.0
Unfinished	0.014 ± 0.002	0.0080	2.4
Planed wood	0.012 ± 0.002	0.0032	1.0
Clay tile	0.014 ± 0.003	0.0080	2.4
Brickwork	0.015 ± 0.002	0.012	3.7
Asphalt	0.016 ± 0.003	0.018	5.4
Corrugated metal	0.022 ± 0.005	0.12	37
Rubble masonry	0.025 ± 0.005	0.26	80
Excavated earth channels:			
Clean	0.022 ± 0.004	0.12	37
Gravelly	0.025 ± 0.005	0.26	80
Weedy	0.030 ± 0.005	0.8	240
Stony, cobbles	0.035 ± 0.010	1.5	500
Natural channels:			
Clean and straight	0.030 ± 0.005	0.8	240
Sluggish, deep pools	0.040 ± 0.010	3	900
Major rivers	0.035 ± 0.010	1.5	500
Floodplains:			
Pasture, farmland	0.035 ± 0.010	1.5	500
Light brush	0.05 ± 0.02	6	2000
Heavy brush	0.075 ± 0.025	15	5000
Trees	0.15 ± 0.05	?	?

† A more complete list is given in Ref. 3, pp. 110–113.

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[i] The references cited for the values tabulated above come from: Chow, V.T., Open Channel Hydraulics, McGraw-Hill, New York, NY, 1959.