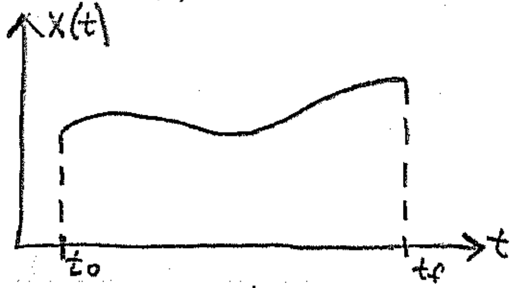
LESSON 1: SIGNALS AND SYSTEMS

OBJECTIVES:
- DEFINE SIGNALS & PROPERTIES
- DEFINE SYSTEMS & PROPERTIES
- LEARN TO DIFFERENTIATE SYSTEM TYPES

SIGNALS: A FUNCTION OR DATA RECORD THAT REPRESENTS A PHYSICAL PHENOMENA

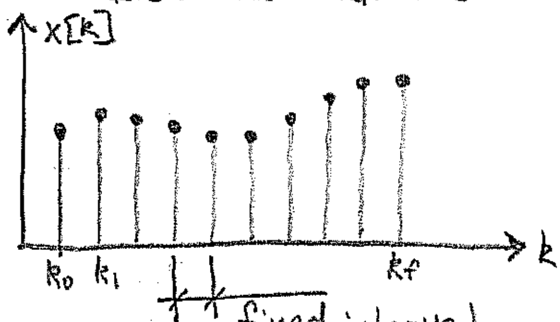
CONTINUOUS-SIGNALS



DOMAIN: (t_0, t_f)

NOT DEFINED OUTSIDE DOMAIN

DISCRETE SIGNALS



fixed interval

DOMAIN: $\{k_0, k_1, \ldots, k_f\}$

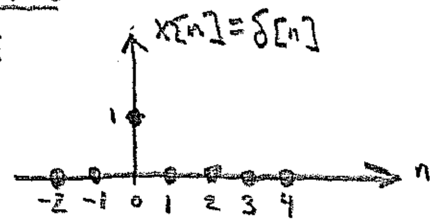
NOT DEFINED BETWEEN SAMPLES

* WE WILL FOCUS ON DISCRETE SIGNALS

→ ELEMENTARY SIGNALS

- UNIT IMPULSE

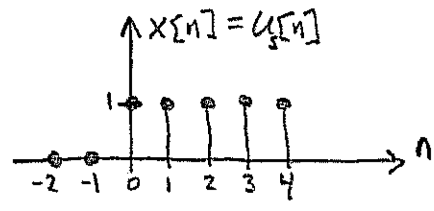
  $\delta[n]$



$$\delta[n] = \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$$

- UNIT STEP

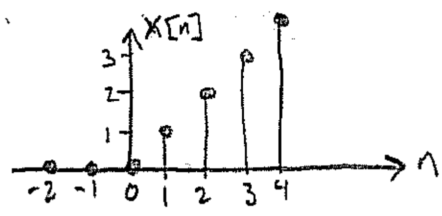
  $u_s[n]$



$$u_s[n] = \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$

- UNIT RAMP

  $u_r[n]$



$$u_r[n] = \begin{cases} n, & \text{for } n > 0 \\ 0, & \text{for } n \leq 0 \end{cases}$$

→ PROPERTIES OF SIGNALS:

- **PERIODICITY:** A signal is periodic if and only if there exists an integer N such that
  $$x[n+N] = x[n] \quad \text{for all integers } n$$

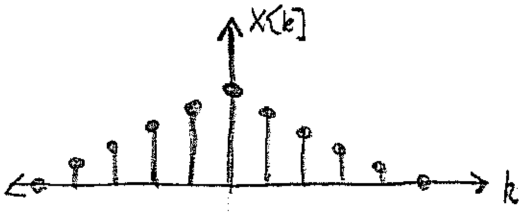
Short form: A signal is periodic $\Leftrightarrow \exists N \in \mathbb{Z}$ s.t. $x[n+N] = x[n] \; \forall n \in \mathbb{Z}$

aperiodic signals: $x[n+N] = -x[n]$

BAD JOKE: ∀∀∃∃

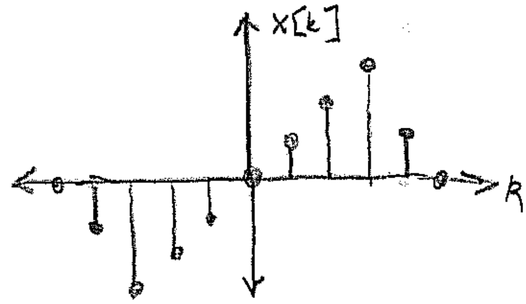
- **SYMMETRY:**

even symmetry



$$x[-n] = x[n] \quad \forall n$$

odd symmetry



$$x[-n] = -x[n] \quad \forall n$$

- **POWER OF A SIGNAL**
  $$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

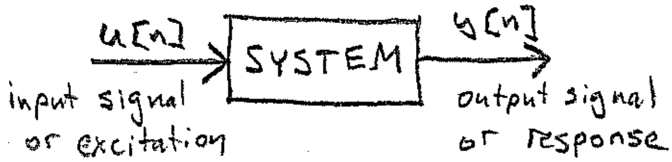
- MORE MANAGABLE FOR SIGNALS OF FINITE LENGTH
- ABSOLUTE VALUE FOR COMPLEX SIGNALS

→ BASIC MANIPULATIONS:

- TIME-SHIFT: $y[n] = x[n-N]$ for $N \in \mathbb{Z}$
- AMPLITUDE SCALING: $y[n] = A\,x[n]$ for $A \in \mathbb{R}$ (or $A \in \mathbb{C}$)
- SUM OF SIGNALS: $y[n] = x_1[n] + x_2[n]$
- PRODUCT OF SIGNALS: $y[n] = x_1[n] \cdot x_2[n]$

SAMPLE-TO-SAMPLE

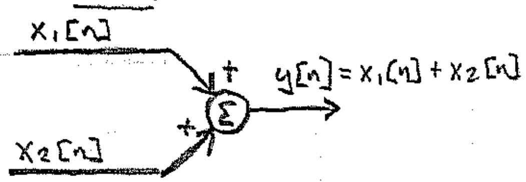
SYSTEMS



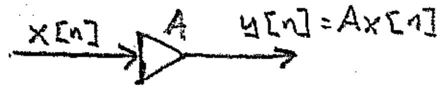
$$y[n] = \mathcal{T}\left(u[n]\right) \qquad u[n] \xrightarrow{\mathcal{T}} y[n]$$

→ BLOCK DIAGRAM REPRESENTATIONS:

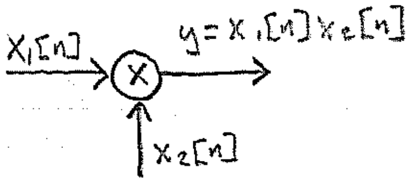
ADDER



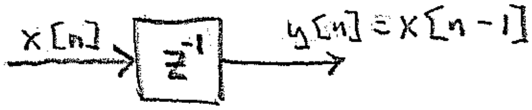
CONSTANT MULTIPLIER



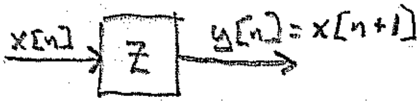
SIGNAL MULTIPLIER



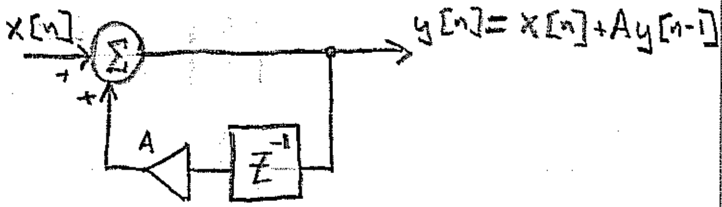
UNIT DELAY



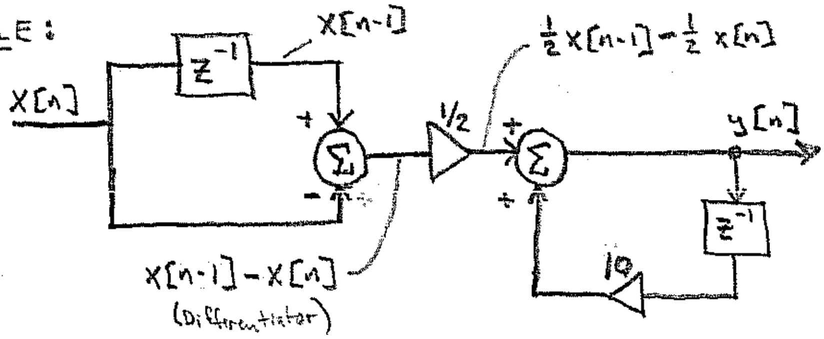
UNIT ADVANCE



FEEDBACK LOOP



EXAMPLE:



$$y[n] = \tfrac{1}{2}x[n-1] - x[n] + 10\,y[n-1]$$

→ TYPES OF SYSTEMS

- MEMORYLESS V. WITH MEMORY:

MEMORYLESS: CURRENT OUTPUT DOES NOT DEPEND ON PAST

$$y[n] = a\,x[n]$$
$$y[n] = n\,x[n] + 5\,x[n]\,x[n]$$

WITH MEMORY: CAN DEPEND ON PAST BEHAVIOR

$$\left.\begin{aligned} y[n] &= x[n] + 3x[n-1] \\ y[n] &= \sum_{k=0}^{m} a_k y[n-k] \end{aligned}\right\} \text{FINITE MEMORY (FIR)}$$

$$y[n] = \sum_{k=0}^{\infty} a_k x[n-k] + \sum_{l=0}^{\infty} b_l y[n-l] \Big\} \text{INFINITE MEMORY (IIR)}$$

- TIME-VARYING V. TIME INVARIENT:

A system is time-invarient $\Leftrightarrow$ $x[n] \xrightarrow{T} y[n]$ implies $x[n-k] \xrightarrow{T} y[n-k]$ $\forall k \in \mathbb{Z}$

Time-Varying systems:

$$y[n] = n\,x[n]$$
$$y[n] = x[-n]$$
$$y[n] = x[n]\cos(\omega_0 n)$$
$$y[n] = \begin{cases} x[n] & \text{for } n \geq 2 \\ 2x[n], & \text{for } n < 2 \end{cases}$$

Time-invarient systems:

$$y[n] = a\,x[n]$$
$$y[n] = x[n] - x[n-1]$$

- LINEAR V. NONLINEAR SYSTEMS:

A system is linear $\Leftrightarrow$

$$T\big(a_1 x_1[n] + a_2 x_2[n]\big) = a_1 T\big(x_1[n]\big) + a_2 T\big(x_2[n]\big) \quad \forall x_1 \,\&\, x_2$$

LINEAR:

$$y[n] = \sum_{k=0}^{m} a_k x[n-k]$$
$$y[n] = n\,x[n]$$

NON-LINEAR:

$$y[n] = x[n]\,x[n]$$
$$y[n] = e^{x[n]}$$

EXAMPLE LINEARITY TEST: $y[n] = x[n]\,x[n]$

$$\rightarrow T\big[a_1 x_1[n] + a_2 x_2[n]\big] = \big(a_1 x_1[n] + a_2 x_2[n]\big)\big(a_1 x_1[n] + a_2 x_2[n]\big)$$
$$① \rightarrow a_1^2 x_1^2[n] + 2a_1 a_2 x_1[n] x_2[n] + a_2^2 x_2^2[n]$$
$$\rightarrow a_1 T[x_1[n]] + a_2 T[x_2[n]] = a_1 x_1^2[n] + a_2 x_2^2[n] \leftarrow ②$$

$$① \neq ② \quad \therefore \; \underline{\text{NOT LINEAR}}$$

- CAUSAL v. NON-CAUSAL:

  A system is causal $\iff$ its output is a function of only past and present information

  CAUSAL: $$y = \sum_{k=0}^{\infty} a_k x[n-k] + \sum_{\ell=0}^{\infty} b_\ell y[n-\ell]$$

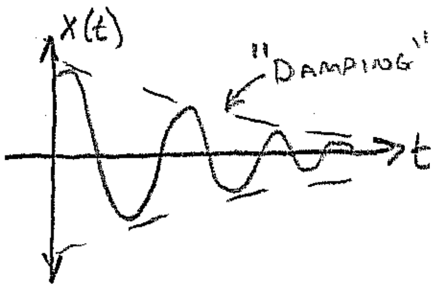
  NON-CAUSAL: $$y[n] = a x[n+2]$$

  $$y[n] = \sum_{i=-m}^{m} x[n-k]$$

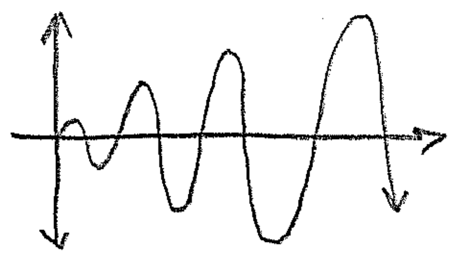
- STABLE v. UNSTABLE:

  BIBO STABILITY — BOUNDED INPUT ALWAYS YIELDS BOUNDED OUTPUT

RESPONSE TO DISTURBANCE



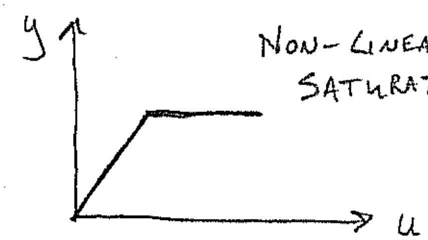
STABLE



UNSTABLE

- A WORD ABOUT SYSTEM INPUTS:
  - → SOME SYSTEM BEHAVIORS REQUIRE CERTAIN INPUTS TO MANIFEST
  - → WITHOUT NECESSARY EXCITATION, NO EVIDENCE IN DATA

e.g.)



NON-LINEAR SATURATION FUNCTION

ALSO: FREQ. DEPENDENT PROPERTIES

* SOURCE FOR LESSON 1: Proakis and Manolakis, "Digital Signal Processing: Principles, Algorithms, and Applications," Prentice Hall, NJ, USA, 1996.