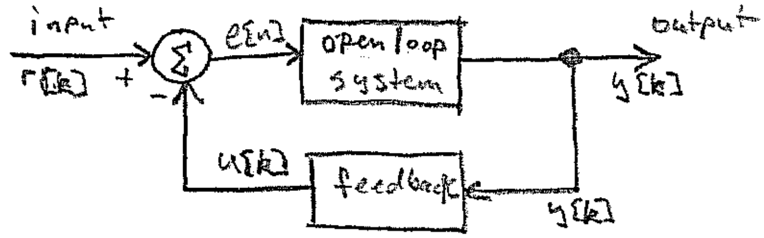
LESSON 13 FEEDBACK SYSTEMS:

OBJECTIVES: • Characterize systems with feedback

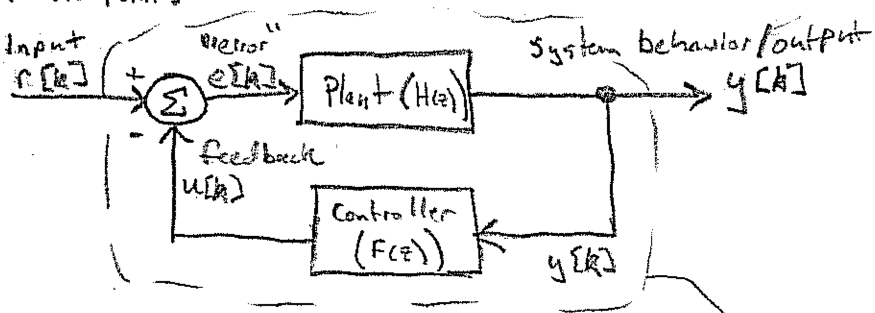
→ Feedback is an important characteristic of many systems.

→ Our ability to determine feedback properties depends on our measurements.

Basic Structure:



Control Viewpoint:



OPEN-LOOP BEHAVIOR:

$$G_{OL}(z) = \frac{Y(z)}{R(z)} = H(z)$$

CLOSED-LOOP BEHAVIOR:

$$Y(z) = H(z)\,E(z)$$

where $E(z) = R(z) - U(z) = R(z) - F(z)\,Y(z)$

$$Y(z) = H(z)\left(R(z) - F(z)\,Y(z)\right)$$

$$Y(z) = H(z)\,R(z) - H(z)\,F(z)\,Y(z)$$

$$H(z)\,R(z) = Y(z) + H(z)\,F(z)\,Y(z)$$

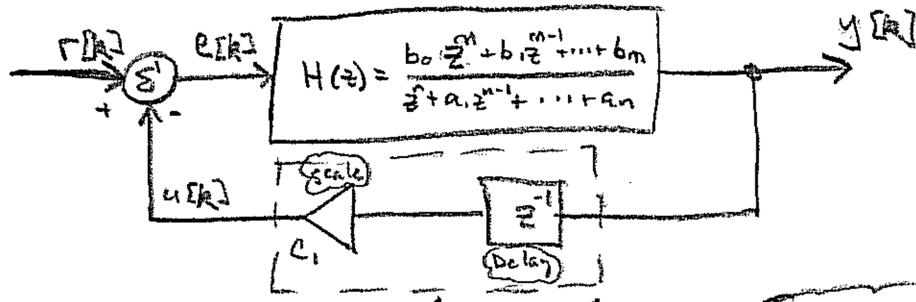
$$H(z)\,R(z) = Y(z)\left(1 + H(z)\,F(z)\right)$$

$$\therefore \boxed{G_{CL}(z) = \frac{Y(z)}{R(z)} = \frac{H(z)}{1 + H(z)\,F(z)}}$$

→ If $r[n]$ & $y[n]$ are the only measurements, we will only be able to characterize $G_{CL}(z)$, not $H(z)$ & $F(z)$!

— Measuring $y[k]$, $r[k]$, AND $e[k]$

EXAMPLE: Delay & scale output (Proportional control)



$\uparrow F(z) = \frac{c_1}{z}$

NOTE: $e[k] = r[k] - u[k]$

2 step process:

1) OPEN-LOOP SYSTEM → use $e[k]$ & $y[k]$ to find $H(z) = \frac{Y(z)}{E(z)}$

$$\hat{y}[k] = -a_1 x[k-1] - \cdots a_n x[k-n] + b_0 e[k] + \cdots + b_m e[k-m]$$

$$\left[\Phi_{OL}\right]\left[\hat{\Theta}_{OL}\right] = \left[b_{OL}\right]$$

$$\begin{bmatrix} & & \vdots & & \\ -y[k-1] \cdots & -y[k-n] & u[k] & \cdots & u[k-m] \\ & & \vdots & & \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \\ b_0 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} \vdots \\ y[k] \\ \vdots \end{bmatrix}$$

↳ yields a's & b's

2) FEEDBACK SYSTEM → use $y[k]$, $r[k]$, and $e[k]$ to find $F(z) = \frac{U(z)}{Y(z)}$

$$\left[\Phi_{FB}\right]\left[\hat{\Theta}_{FB}\right] = \left[b_{FB}\right]$$

$$\begin{bmatrix} \vdots \\ y[k] \\ \vdots \end{bmatrix} \left[c_1\right] = \begin{bmatrix} \vdots \\ r[k] - e[k] \\ \vdots \end{bmatrix}$$

$e[k] = r[k] - u[k]$
$e[k] = r[k] - c\,y[k]$
$c\,y[k] = r[k] - e[k]$

↑ yields c_1

→ Can easily generalize to $F(z) = \frac{c_0 z^p + c_1 z^{p-1} + \cdots + c_p}{z^q + d_1 z^{q-1} + \cdots + d_q}$