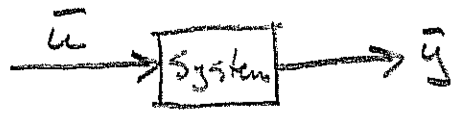
Lesson 14 – MIMO v. SISO

OBJECTIVES: Introduce MIMO system ID



Recall: $\bar{u}$ & $\bar{y}$ may be vectors.

Terminology:

Inputs: SI – Single-Input → SISO

MI – Multi-Input → SIMO

Outputs: SO – Single-Output → MISO

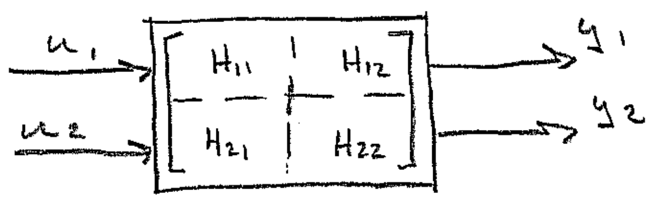
MO – Multi-Output → MIMO

R = # inputs

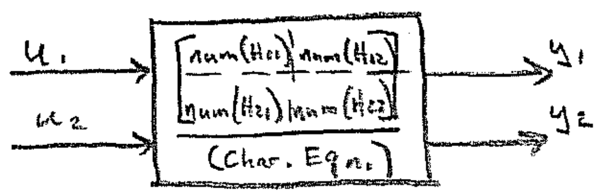
S = # outputs

$$H(\omega) \text{ or } H(z) \longrightarrow \bar{H}(\omega) \text{ or } \bar{H}(z)$$

e.g.)



- In transfer function matrix, all transfer functions share the same poles.
  - Char. Eqn. same
  - Zeros are different



- State-space more elegant representation.

MIMO:

Ex. Linear-Difference Eqns. 2 Input, 2 Output System

for y_1, set $y_2 = 0$: → n & m constant (same for all inputs/outputs)

$$\hat{y}_1[k] = -a_1 y_1[k-1] - \cdots - a_n y_1[k-n] + b_{11,0} u_1[k] + \cdots + b_{11,m} u_1[k-m] + b_{12,0} u_2[k] + \cdots + b_{12,m} u_2[k-m] + e_1[k]$$

for y_2, set $y_1 = 0$:

$$\hat{y}_2[k] = -a_1 y_2[k-1] - \cdots - a_n y_2[k-n] + b_{21,0} u_1[k] + \cdots + b_{21,m} u_1[k-m] + b_{22,0} u_2[k] + \cdots + b_{22,m} u_2[k-m] + e_2[k]$$

→ Matrix Form:

$$\hat{\bar{y}}[k] = \bar{\phi}_{k-1} \hat{\bar{\theta}}$$

$$\underbrace{\begin{bmatrix} y_1[k] \\ y_2[k] \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} -y_1[k-1] \cdots -y_1[k-n] & u_1[k] \cdots u_1[k-m] \; u_2[k] \cdots u_2[k-m] & [0 \cdots 0]_{2(m+1)} \\ -y_2[k-1] \cdots -y_2[k-n] & [0 \cdots 0]_{2(m+1)} & u_1[k] \cdots u_1[k-m] \; u_2[k] \cdots u_2[k-m] \end{bmatrix}}_{2 \times (n+4(m+1))} \underbrace{\begin{bmatrix} a_1 \\ \vdots \\ a_n \\ b_{11,0} \\ \vdots \\ b_{11,m} \\ b_{12,0} \\ \vdots \\ b_{12,m} \\ b_{21,0} \\ \vdots \\ b_{21,m} \\ b_{22,0} \\ \vdots \\ b_{22,m} \end{bmatrix}}_{(n+4(m+1))\times 1}$$

In general:
$$\hat{\bar{y}}_k \in \mathbb{R}^{S\times 1}$$
$$\bar{\phi}_k \in \mathbb{R}^{S\times (n+R^2(m+1))}$$
$$\bar{\theta} \in \mathbb{R}^{(n+R^2(m+1))\times 1}$$

→ Least-Squares Soln:

$$\bar{b} = \bar{\bar{\Phi}} \hat{\bar{\theta}}$$ ← looks same as 5150!

define $p = \max(m, n)$, N = num samples

$$\begin{bmatrix} \bar{y}[p+1] \\ \vdots \\ \bar{y}[k+1] \\ \vdots \\ \bar{y}[N] \end{bmatrix} = \begin{bmatrix} \bar{\phi}_p \\ \vdots \\ \bar{\phi}_k \\ \vdots \\ \bar{\phi}_{N-1} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \\ b_{11,0} \\ \vdots \\ b_{22,m} \end{bmatrix}$$

$$\hat{\theta} \in \mathbb{R}^{(n+R^2(m+1))\times 1}$$
$$\bar{b} \in \mathbb{R}^{S(N-p)\times 1}$$
$$\bar{\bar{\Phi}} \in \mathbb{R}^{S(N-p)\times (n+R^2(m+1))}$$

Class Discussion