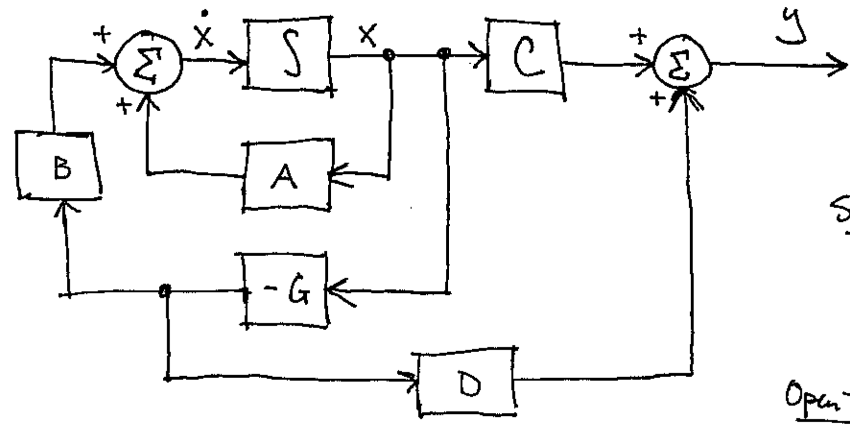
LESSON 22 - Kalman Filter

- objectives :
  - Introduce state-space control
  - Introduce state-space estimation

State-Space Control:



State feedback

$u = -Gx$

Open-loop poles

Eigenvalues of (A)

Closed-loop poles

Eigenvalues of $(A - BG)$

→ We can arbitrarily assign poles for controllable (A, B).
- Theory says ok
- Reality says we only have limited control ability

→ LQR control:

→ Tradeoff between performance and control effort.
- Choose G that minimizes cost function:

$$J(u) = \int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

for "optimal" controller.
- poles of $(A - BG)$ should be stable

STATE ESTIMATION: (Text, Ch. 5)

- Similar to feedback control.
  - Control error dynamics.
  - Drive error to zero.
  - Error dynamics must also be stable.

Remember observer Markov derivation

$$\hat{x}[k+1] = A\hat{x}[k] + Bu[k] + K(y[k] - \hat{y}[k])$$
$$\hat{y}[k] = C\hat{x}[k] + Du[k]$$

$\to$ error dynamics: $e[k+1] = (A+KC)e[k]$

Choose K such that:

$$\lim_{k\to\infty} e[k] \doteq 0$$

How fast should $e[k] \to 0$?

- Depends on amount of noise & disturbance in signal.

Reality
$$x[k+1] = Ax[k] + Bu[k] + V[k]$$
$$y[k] = Cx[k] + Du[k] + W[k]$$

Steady-state Kalman Filter:

- Suppose Noise & Disturbance statistics are very well known:

$E[w[k]] = 0$, $E[V[k]] = 0$ } white noise

$E[ww^T] = Q$ Noise Covariance Matrix

$E[VV^T] = R$ Disturbance Covariance matrix

$E[WV^T] = S$ Cross Covariance Matrix

The Covariance Matricies Function in a similar way to the weighting matricies in LQR control.

Minimize: $P = \lim_{t\to\infty} E\left[(x-\hat{x})(x-\hat{x})^T\right]$

For "optimal" estimator.

Steady-State Kalman Gain:

$$K = (APC^T + S)(CPC^T + R)^{-1}$$

where P is solution to algebraic Riccati equation:

$$P = APA^T - (APC^T + S)(CPC^T + R)^{-1}(APC^T + S)^T + Q$$

↳ solve numerically (Matlab)

- Resulting estimator system:

$$\hat{x}[k+1] = (A - KC)\hat{x}[k] + Bu[k] + Ky[k]$$

- <u>KALMAN FILTER</u>: updates Kalman gain depending on error dynamics observed.

  → This error is assumed to be due to v & w.

$$K[k] = [A\,P[k]C^T + S][C\,P[k]C^T + R]^{-1}$$

$$P[k+1] = A\,P[k]A^T - K[k][C\,P[k]C^T + R](K[k])^T + Q$$

Noise statistics: