LESSON 9: DT Transfer Function Models:

OBJECTIVES:
- CANONICAL FORMS
- INPUT/OUTPUT v. OUTPUT-ONLY

LINEAR DIFFERENCE EQN:

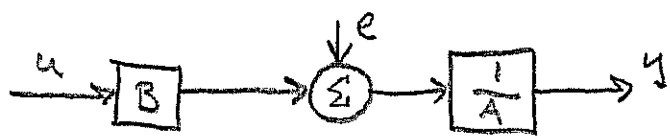
$$\hat{y}[k] = \text{estimate of } y[k]$$

$$\hat{y}[k] = -a_1 y[k-1] - a_2 y[k-2] - \cdots - a_n y[k-n] + b_0 u[k] + b_1 u[k-1] + \cdots + b_m u[k-m] + \underline{e[k]}$$

$$e[k] = y[k] - \hat{y}(k)$$

IN Z-DOMAIN: $$\frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} + E(z)$$

THIS IS A CLASSIC ARX MODEL FORM
→ Auto-Regressive with eXogenous input



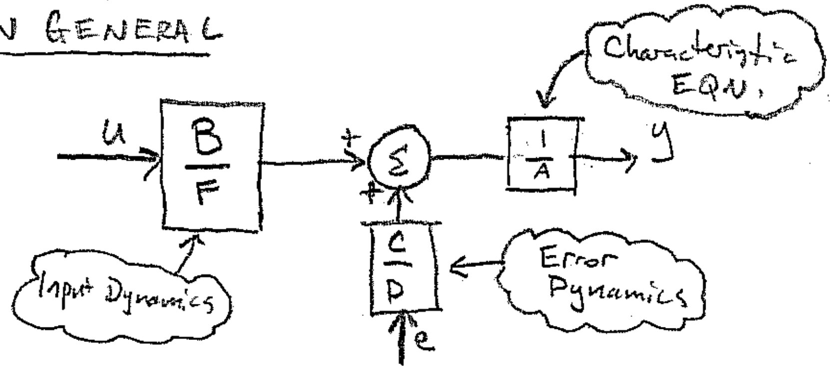
$$A(z^{-1})\,Y(z) = B(z^{-1})\,U(z) + E(z)$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$$

→ Here, we have a model that considers the dynamics of the system $(\frac{1}{A})$ and the input (B).

→ Depending on what amount of freedom we grant to our model, we will get different model classes.

– IN GENERAL



$$A(z^{-1})\,Y(z) = \frac{B(z^{-1})}{F(z^{-1})}U(z) + \frac{C(z^{-1})}{D(z^{-1})}E(z)$$

$$Y(z) = \frac{B(z^{-1})}{A(z^{-1})\,F(z^{-1})}U(z) + \frac{C(z^{-1})}{A(z^{-1})\,D(z^{-1})}E(z)$$

	System	Error	Input
AR terms:	$A(z^{-1})$	$D(z^{-1})$	$F(z^{-1})$
MA terms: (moving average)			$C(z^{-1})$
Exogenous input terms:		$B(z^{-1})$	

→ When $B = 0$, output-only model.

→ When D and/or B $\neq 0$ consider error dynamics.

(Implementation is complicated)

(see model family handout.)

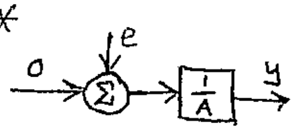
→ We will discuss: → ARX
→ AR / ARMA

Generalized Structure

$$A(z)Y(z) = \frac{B(z)}{F(z)}U(z) + \frac{C(z)}{D(z)}E(z)$$

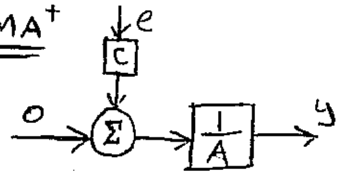
Output Only

AR*



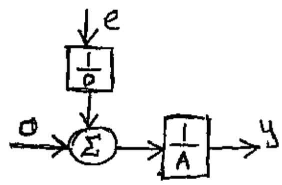
$$A(z)Y(z) = E(z)$$

ARMA+



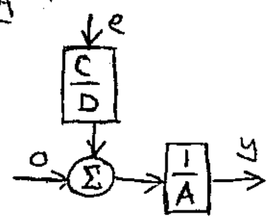
$$A(z)Y(z) = C(z)E(z)$$

ARAR*



$$A(z)Y(z) = \frac{1}{D(z)}E(z)$$

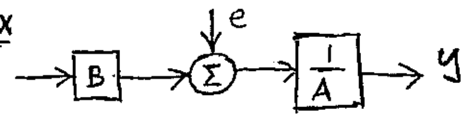
ARARMA*+



$$A(z)Y(z) = \frac{C(z)}{D(z)}E(z)$$

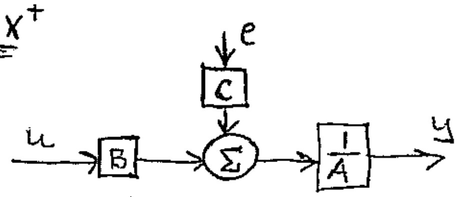
Input/Output

ARX



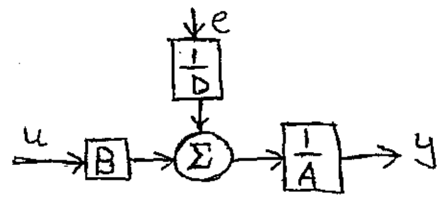
$$A(z)Y(z) = B(z)U(z) + E(z)$$

ARMAX+



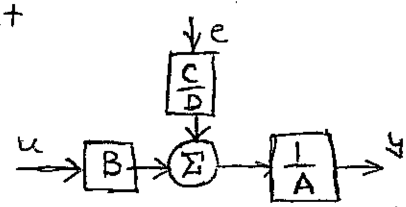
$$A(z)Y(z) = B(z)U(z) + C(z)E(z)$$

ARARX



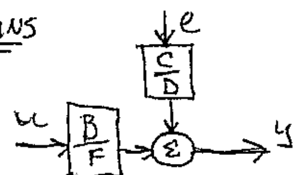
$$A(z)Y(z) = B(z)U(z) + \frac{1}{D(z)}E(z)$$

ARARMAX+



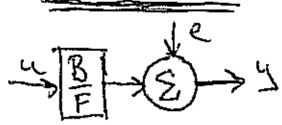
$$A(z)Y(z) = B(z)U(z) + \frac{C(z)}{D(z)}E(z)$$

Box-Jenkins



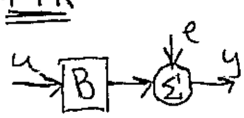
$$Y(z) = \frac{B(z)}{F(z)}U(z) + \frac{C(z)}{D(z)}E(z)$$

Output Error



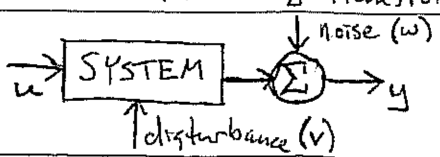
$$Y(z) = \frac{B(z)}{F(z)}U(z) + E(z)$$

FIR



$$Y(z) = B(z)U(z) + E(z)$$

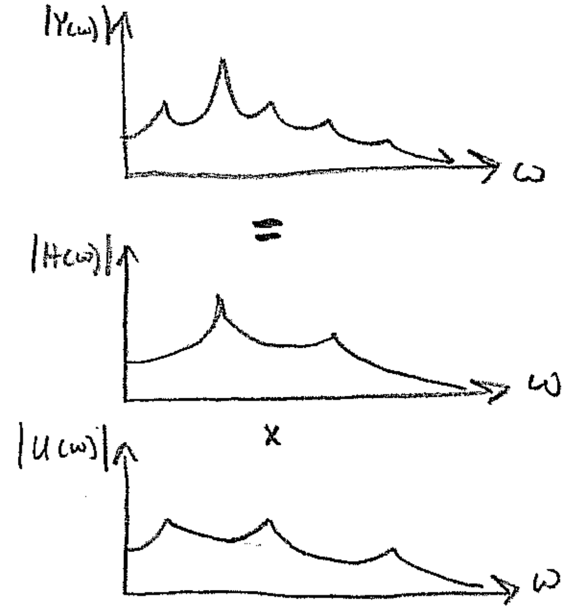
$Y(z)$ = Output Z-transform
$U(z)$ = Input Z-transform
$E(z)$ = Disturbance/Noise/Error Z-Transform



* Not commonly used (Ljung)
\+ Must be implemented as a filter

"OUTPUT-ONLY" SYSTEMS:

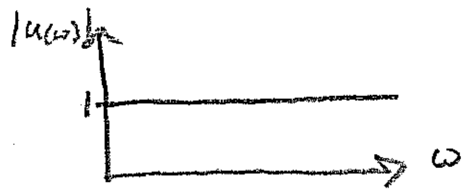
FREQ.-DOMAIN CONVOLUTION:



- We can only recover $H(\omega)$ from $Y(\omega)$ if $U(\omega)$ is known (same idea for $Y(z)$).

- Certain inputs are more useful:

→ $u[k] = \delta[k]$
$U(z) = 1$
$U(\omega) = 1$



$$Y(\omega) = H(\omega)U(\omega) = H(\omega)\,1 = H(\omega)$$

→ $u[k]$ = white noise realization
$$\left.\begin{matrix} U(z) = 1 \\ U(\omega) = 1 \end{matrix}\right\} Y(\omega) = H(\omega)$$
(operational modal analysis)

→ otherwise, try to recover true Characteristic Equation ($A(\omega)$) by modeling error dynamics.
→ ARMA