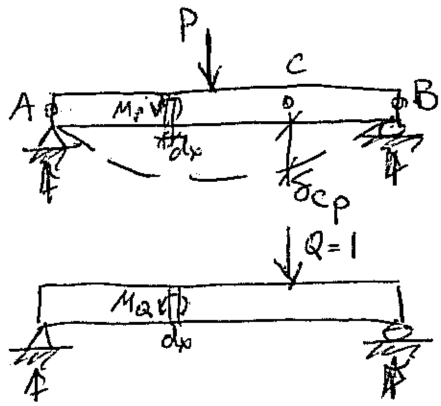
LESSON 14 – VIRTUAL WORK – BEAMS



P-SYSTEM

Q-SYSTEM

IMPORTANT CHANGE FROM TRUSSES:

INTERNAL FORCE CHANGES WITHIN THE MEMBER!
∴ We must integrate to find internal strain energy.

WORK

$$W_Q = Q\delta_P$$

STRAIN ENERGY

$$dU_Q = M_Q\, d\theta$$

where $d\theta = \dfrac{M_P}{EI}\, dx$

$$\text{so } U_Q = \int_{x=0}^{x=L} dU_Q$$

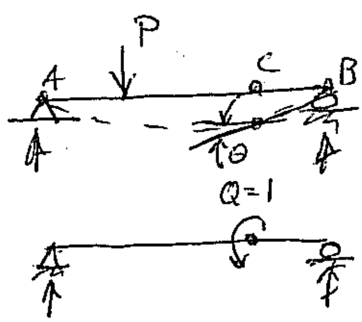
$$U_Q = \int_{x=0}^{x=L} M_Q \frac{M_P}{EI}\, dx$$

$$W_Q = U_Q$$

$$\boxed{Q\delta_P = \int_{x=0}^{x=L} M_Q \frac{M_P}{EI}\, dx}$$

What about Rotation (slope)?

APPLY A UNIT MOMENT:



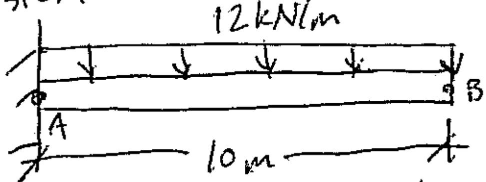
$$\boxed{Q\theta_P = \int_{x=0}^{x=L} M_Q \frac{M_P}{EI}\, dx}$$

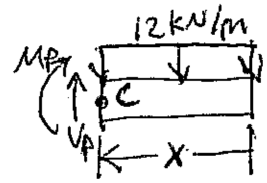
EXAMPLE.

a.) FIND DISP. AT B DUE TO LOAD

$E = 200\ GPa$, $I = 500 \cdot 10^{-6}\ m^4$

P-SYSTEM

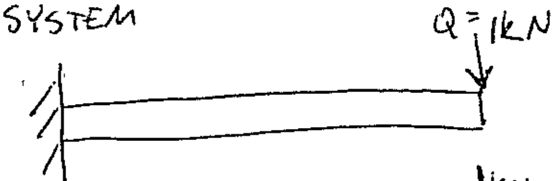


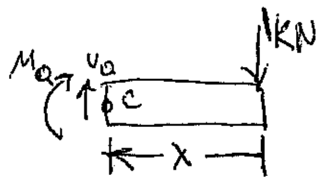


$\circlearrowleft + \ \Sigma M_C = 0 \rbrack \ -M_P - 12x\left(\frac{x}{2}\right) = 0$

$M_P = -6x^2\ (kN/m)$

Q-SYSTEM





$\circlearrowleft + \ \Sigma M_C = 0 \rbrack \ -M_Q - 1(x) = 0$

$M_Q = -x\ (kN)$

$$Q\,\delta_B = \int_0^L M_Q \frac{M_P}{EI}\,dx$$

$$1 \cdot \delta_B = \int_0^{L=10m} \frac{(-x)(-6x^2)}{EI}\,dx$$

$$(1\,kN)\,\delta_B = \left[\frac{6x^4}{4EI}\,(kN^2/m) \right|_{0m}^{10m} \Bigg]$$

$$(1\,kN)\,\delta_B = \frac{6\,(10m)^4\,(kN^2/m)\,(1000\ N/kN)}{4\,(200 \cdot 10^9\ \frac{N}{m^2})(500 \cdot 10^{-6}\ m^4)}$$

$$\delta_B = 0.150\ m$$

$$\boxed{\delta_B = 150\ mm}$$

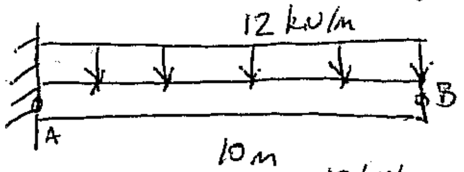
NOTE THE DEFINITE INTEGRAL

INTEGRAL TYPE	ANSWER	SPECIAL FEATURE
Definite	Number	Limits of Integration
Indefinite	Function	Constants of Integration

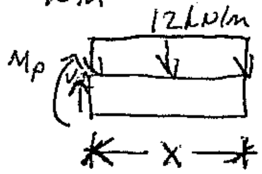
... EXAMPLE CONTINUED

b.) FIND SLOPE AT B

P-SYSTEM

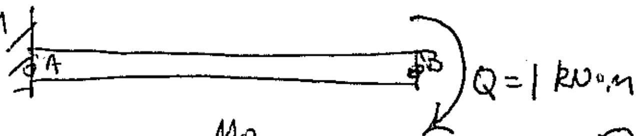


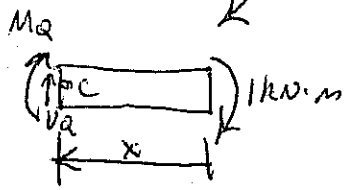
(UNCHANGED FROM PART a)



$$M_P = -6x^2 \ (\text{kN/m})$$

Q-SYSTEM





$$\circlearrowleft^{+} \sum M_C = 0 \Big] \quad -M_Q - 1\,\text{kN·m} = 0$$

$$M_Q = -1\ \text{kN·m}$$

$$Q\,\theta_B = \int_0^L M_Q \frac{M_P}{EI}\,dx$$

$$(1\ \text{kN·m})\,\theta_B = \int_{0\text{m}}^{10\text{m}} \frac{(-1)(-6x^2)}{EI}\,dx$$

$$(1\ \text{kN·m})\,\theta_B = +\frac{2x^3}{EI}(\text{kN}^2)\Big|_{0\text{m}}^{10\text{m}}$$

$$(1\ \text{kN·m})\,\theta_B = \frac{2(10\text{m})^3(\text{kN}^2)(1000\ \text{N/kN})}{(200\cdot10^9\ \text{N/m}^2)(500\cdot10^{-6}\ \text{m}^4)}$$

$$\boxed{\theta_B = 0.02\ \text{rad.}}$$

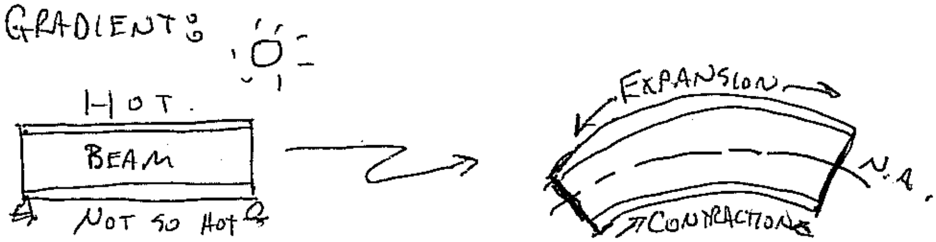
OTHER EFFECTS IN BEAMS

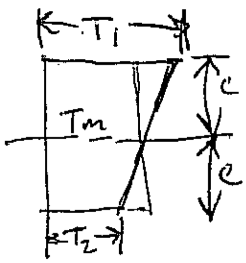
→ **AXIAL LOAD:**
BEAMS THAT ARE PART OF FRAMES CAN CARRY AXIAL LOAD

STRAIN ENERGY LOOKS JUST LIKE TRUSS STRAIN ENERGY:

$$\boxed{Q\delta = \frac{F_Q F_P L}{AE}}$$

→ **Temperature GRADIENT:**





$T_m = \frac{T_1 + T_2}{2}$

$\Delta T_m = T_1 - T_m = T_m - T_2$

α = Coefficient of thermal expansion

C = mid-depth of beam

$$d\theta = \frac{\alpha(\Delta T_m)}{c}\,dx$$

$$\therefore \boxed{Q\delta = \int_0^L M_Q \frac{\alpha(\Delta T_m)}{c}\,dx}$$

COMBINE THESE EFFECTS WITH LOAD EFFECTS FOR DEFLECTION OF BEAM DUE TO ALL STIMULI!