Lesson 16 - Final Thoughts on Energy Methods

\[ \text{Non-Prismatic Elements} \]

**Def:** Prismatic members = Members with constant cross-section along length

When members are non-prismatic, \( I \) becomes \( I(x) \) and must become part of the integral.

Tapered Beam:

\[ \int_0^L \frac{M_0(x)M_0(x)}{EI(x)} \, dx \]

Stepped Beam:

\[ \int_0^{L_1} \frac{M_0(x)M_0(x)}{EI_1} \, dx + \int_{L_1}^{L} \frac{M_0(x)M_0(x)}{EI_2} \, dx \]

\[ I_1 \]

\[ I_2 \]

\[ \lambda \]

\[ \lambda \]

Finite Summation - Approximate Method for Non-Prismatic Members

\[ SQ = \sum_{i=1}^{N} \frac{M_0M_0}{EI} \Delta x_i \]

\( S = \# \text{ elements} \)

As \( N \to \infty \), \( \Delta x \to 0 \) we approach exact solution.

In principle, this is the idea behind many finite element method approaches used in civil engineering.
<table>
<thead>
<tr>
<th>STRUCTURE TYPE</th>
<th>Wq</th>
<th>Uq Axial</th>
<th>Uq Axial</th>
<th>Uq BUCK-Temp Change</th>
<th>Uq TEMP GEAR</th>
<th>Uq FAB, EVC/CHARGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUSSES</td>
<td>QS</td>
<td>( \frac{F_0 F_a L}{AE} )</td>
<td>( N = # \text{Members} )</td>
<td>( \sum \Delta \theta )</td>
<td>( 0 )</td>
<td>( \varepsilon F_a (\Delta L) )</td>
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<td></td>
<td>No Moment in trusses</td>
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<tr>
<td>BEAMS</td>
<td>QS</td>
<td>( \frac{F_0 F_a L}{AE} )</td>
<td>( \int_0^L \frac{M_a M_p}{EI} dx )</td>
<td>( F_a \Delta L )</td>
<td>( \frac{N^T \Delta T_a M_t}{\alpha C} )</td>
<td>( F_a (\Delta L) )</td>
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<td>Often very small</td>
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<td>Usually = 0</td>
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<tr>
<td>FRAMES</td>
<td>QS</td>
<td>( \frac{N^T F_0 F_a L}{EI} )</td>
<td>( \sum \frac{M_a M_p}{EI} dx )</td>
<td>( F_a \Delta L )</td>
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<td>affected by temp. change</td>
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</tbody>
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Q = Dummy Load
\( \delta \) = Disp. At Q
\( \theta \) = Direction of FQ
Maxwell-Betti Law of Reciprocal Deformations

- Text derives it using virtual work
- Fundamental structural theorem

Requisites:
1) Stable structure
2) Elastic structure
3) No support movement
4) No temp. change

Statement:
A deflection produced at a point A, due to a unit load at point B, is equal in magnitude to the displacement at point B, due to the unit load at point A.

\[ \delta_{AB} = \delta_{BA} \]

True for forces & displacements in beams, frames, & trusses.
Also true for moments & rotations in beams & frames.

Will look at the statement of reciprocity again in two weeks in the analysis of indeterminate structures.