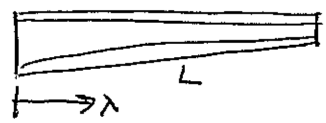
LESSON 16 – FINAL THOUGHTS ON ENERGY METHODS

→ NON-PRISMATIC ELEMENTS

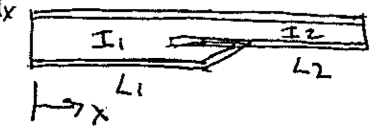
DEF. PRISMATIC MEMBERS: MEMBERS WITH CONSTANT CROSS-SECTION ALONG LENGTH

- When members are non-prismatic I becomes $I(x)$ and must become part of the integral:

TAPERED BEAM $\displaystyle\int_0^L \frac{M_Q(x)\,M_P(x)}{EI(x)}\,dx$

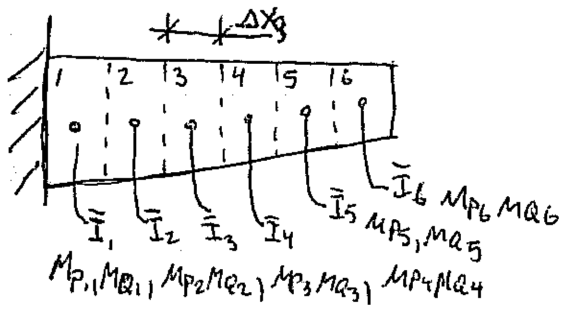


STEPPED BEAM $\displaystyle\int_0^{L_1} \frac{M_Q(x)M_P(x)}{EI_1}dx + \int_{L_1}^{L_2+L_1} \frac{M_Q(x)M_P(x)}{EI_2}dx$



→ FINITE SUMMATION

– APPROXIMATE METHOD FOR NON-PRISMATIC MEMBERS



$$\delta Q = \sum_{i=1}^{N} M_{Q_i} M_{P_i} \frac{\Delta x_i}{EI_i}$$

N = # elements

→ As $N \to \infty$, $\Delta x \to 0$ & we approach exact solution.

→ IN PRINCIPLE, THIS IS THE IDEA BEHIND MANY FINITE-ELEMENT METHOD APPROACHES USED IN CIVIL ENGINEERING.

BIG SUMMARY OF VIRTUAL WORK

VIRTUAL WORK = W_Q = U_Q = STRAIN ENERGY

STRUCTURE TYPE	W_Q	U_Q AXIAL	U_Q FLEXURE	U_Q BULK TEMP CHANGE	U_Q TEMP GRAD	U_Q FAB. ERROR / CAMBER
TRUSSES	$Q\delta$	$\sum_{i=1}^{N} \frac{F_Q F_P L}{AE}$ N = # Members	0 No Moment in trusses	$\sum_{i=1}^{N} F_Q \alpha \Delta T L$ N = # members affected by temp. change.	0	$\Sigma F_Q (\Delta L)$
BEAMS	$Q\delta$	$\frac{F_Q F_P L}{AE}$ often very small	$\int_0^L M_Q \frac{M_P}{EI} dx$	$F_Q \alpha \Delta T L$ usually = 0	$\int_0^L \frac{M_Q \alpha (\Delta T_m)}{c} dx$	$F_Q (\Delta L)$ usually = 0
FRAMES	$Q\delta$	$\sum_{i=1}^{N} \frac{F_Q F_P L}{AE}$ often very small N = # Members	$\sum_{i=1}^{N} \int_0^L \frac{M_Q M_P}{EI} dx$ N = # Members	$\sum_{i=1}^{N} F_Q \alpha \Delta T L$ N = # Members affected by temp. change	$\Sigma \int_0^L \frac{M_Q \alpha (\Delta T_m)}{c} dx$	$\Sigma F_Q (\Delta L)$

Q = DUMMY LOAD

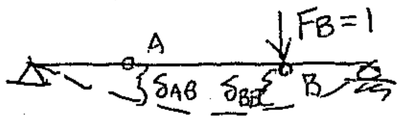
δ = DISP. AT Q, IN DIRECTION OF Q

MAXWELL-BETTI LAW OF RECIPROCAL DEFORMATIONS

- TEXT DERIVES IT USING VIRTUAL WORK
- FUNDAMENTAL STRUCTURAL THEOREM
- REQUIRES: 1) STABLE STRUCTURE
  2) ELASTIC STRUCTURE
  3) NO SUPPORT MOVEMENT
  4) NO TEMP. CHANGE

STATEMENT:

A DEFLECTION PRODUCED AT A POINT A, DUE TO A UNIT LOAD AT POINT B, IS EQUAL IN MAGNITUDE TO THE DISPLACEMENT AT POINT B, DUE TO THE UNIT LOAD AT POINT A.



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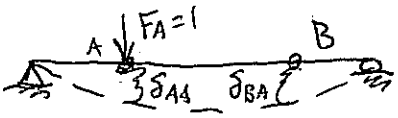

LOAD AT A

$$\delta_{AB} = \delta_{BA}$$

- TRUE FOR FORCES & DISPLACEMENTS IN BEAMS, FRAMES, & TRUSSES. ALSO TRUE FOR MOMENTS & ROTATIONS IN BEAMS & FRAMES.

→ WILL LOOK AT THE STATEMENT OF RECIPROCITY AGAIN IN TWO WEEKS IN THE ANALYSIS OF INDETERMINATE STRUCTURES.