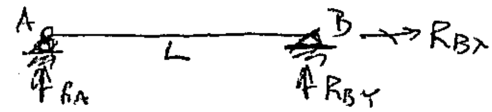
INFLUENCE LINES & MÜLLER-BRESLAU PRINCIPLE

- VERY POWERFUL SHORTCUT FOR INFLUENCE LINES:
  - DERIVED FROM ENERGY METHODS
  - GIVES SHAPE ONLY, MAGNITUDE ESTABLISED BY OTHER MEANS.

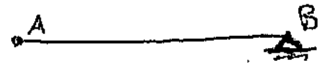
- STATEMENT OF MÜLLER-BRESLAU PRINCIPLE:

The influence line for any reaction or internal force corresponds to the deflected shape of the structure produced by removing the capacity of the structure to carry that force & introducing a unit deformation into the modified structure corresponding to the released restraint.

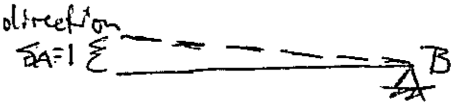
EXAMPLE 1: FIND SHAPE OF INFLUENCE LINE FOR R_A



1) Remove support at A: in Y-direction

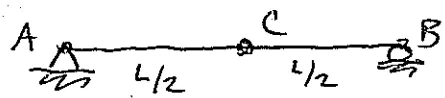


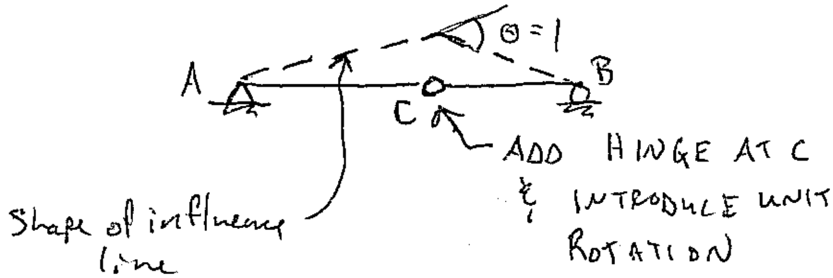
2) Apply a unit displacement in y-direction



Deformed shape is shape of influence line.

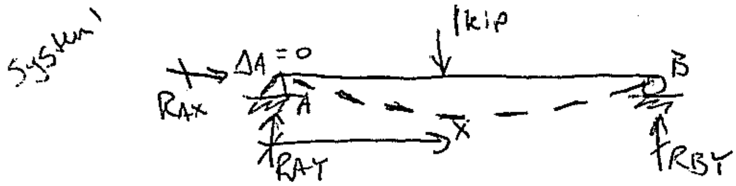
EXAMPLE 2: FIND SHAPE OF INFLUENCE LINE FOR M_C





DERIVATION OF MÜLLER-BRESLAU:

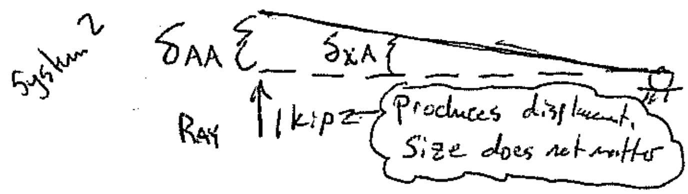
- USE VIRTUAL WORK



CONSIDER INFLUENCE LINE FOR R_{AY}:

Remove Support at A AND APPLY A UNIT Deformation:

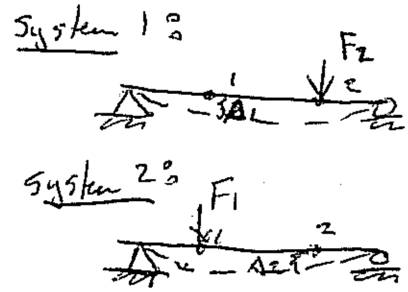
δ_{AA} = Deformation At A due to UNIT Deformation at A



δ_{XA} = Deformation at x due to unit deformation at A

Apply Maxwell-Betti Law of Reciprocal Deformations:

$$\Sigma F_1 \Delta_2 = \Sigma F_2 \Delta_1$$



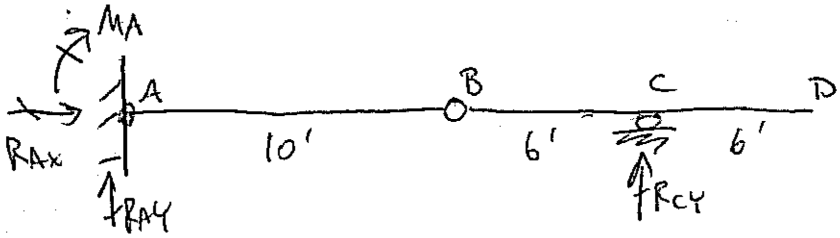
$$R_{AY}\,\delta_{AA} + (1\,kip)\,\delta_{XA} = (1k)\Delta_A \quad (=0,\ \Delta_A = 0)$$

$$\therefore R_{AY} = -\frac{\delta_{XA}}{\delta_{AA}} \quad \begin{matrix}\leftarrow \text{varies} \\ \leftarrow \text{constant}\end{matrix}$$

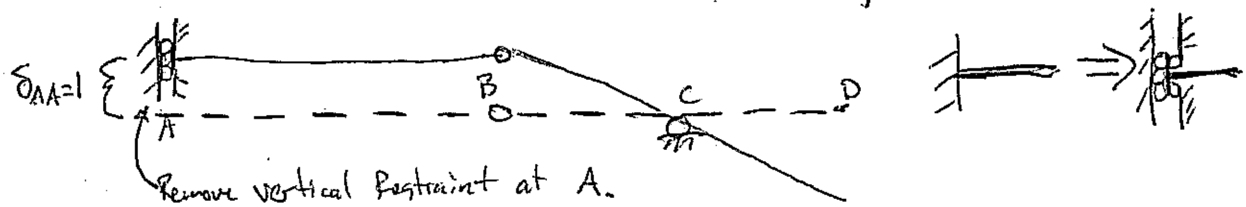
$$R_{AY} = \left(-\frac{1}{\delta_{AA}}\right)\delta_{XA}$$

The influence line is a constant times the deflected shape of the beam. Müller-Breslau method is valid.

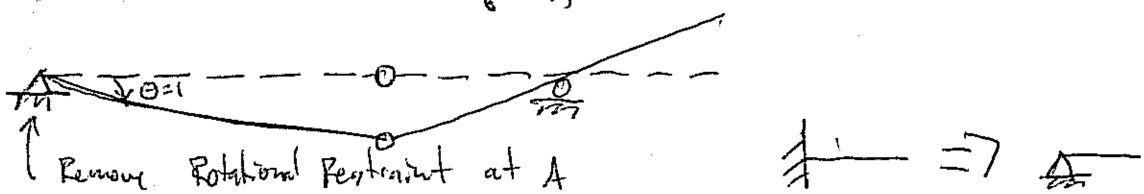
ANOTHER EXAMPLE:



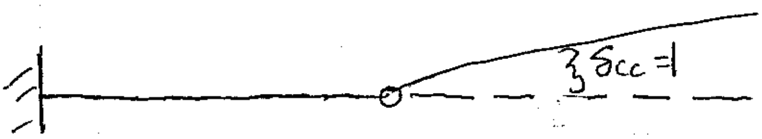
~~DRAW~~ SKETCH INFLUENCE LINE DIAGRAM FOR VERT. RXN. AT A (R_{AY}):



SKETCH INFLUENCE LINE DIAGRAM FOR MOMENT AT A (M_A)



SKETCH INFLUENCE LINE DIAGRAM FOR VERT. RXN. AT C (R_{CY})



What about stability of the reduced support structure?

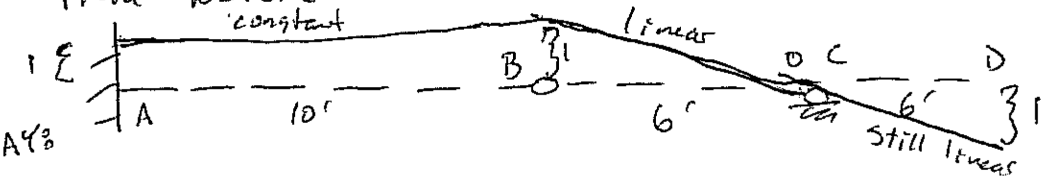
Not stable if the original structure was determinate.

What if we want exact numbers, not just a sketch?

FIND A SINGLE POINT, FILL IN THE REST:

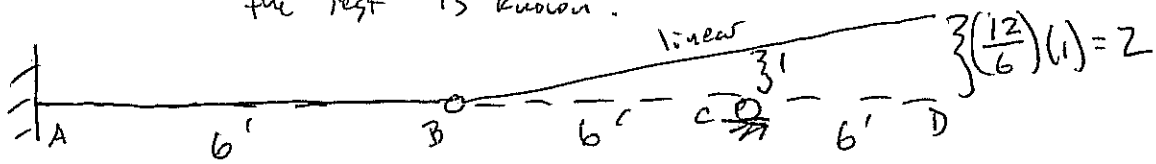
From BEFORE:

R_{AY}:

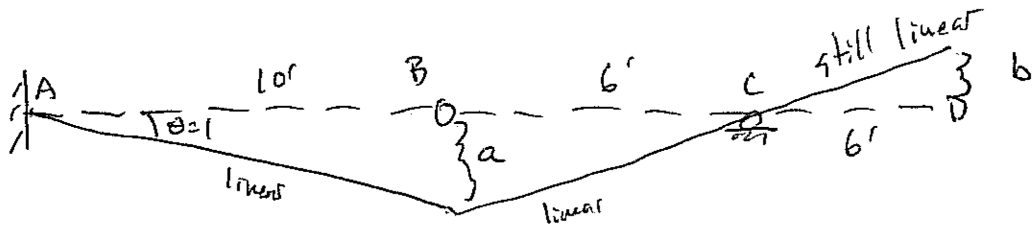


If we find that $R_A = 1$ when unit load is at A, the rest is known.

R_{CY}:



M_A



if a

$b = \frac{a}{6}(6) = a$

Put a unit load at B, solve for M_A

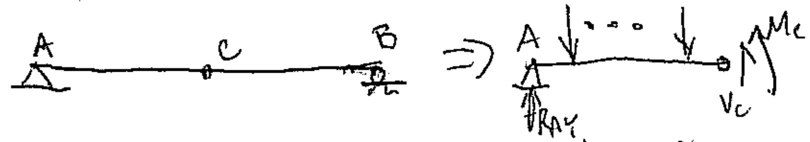
$M_A = -10\text{k}\cdot\text{ft}$

$a = -10\text{k}\cdot\text{ft}$, $b = 10\text{k}\cdot\text{ft}$

What if you are asked for shear?

→ Cut beam at point of interest:

V_C:



generate influence diagram:

→ or use Müller-Breslau

V_C

→ What about shear at ends of beams? Compare to reactions.