LESSON 22: FLEXIBILITY METHOD CONTINUED

THIS LESSON: • FLEXIBILITY METHOD FOR TRUSSES
• MULTIPLE REDUNDANTS

A) APPLICATION TO TRUSSES:

→ FOLLOW THE SAME PROCESS AS USED FOR BEAMS & FRAMES ... USUALLY.

DETERMINACY FOR TRUSSES: $B + R > 2J$: INDETERMINATE
DEGREE $= 2J - (B+R)$

$B = $ # BARS
$R = $ # REACTION
$J = $ # JOINTS

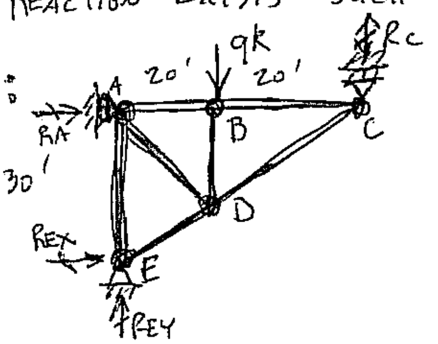
$B + R = 2J$: DETERMINATE / STABLE
$B + R < 2J$: UNSTABLE

(Rules regarding parallel and concurrent force systems apply)

→ 2 Cases for trusses
1) YOU ARE FREE TO REMOVE ONE REACTION WITHOUT CREATING A PARALLEL OR CONCURRENT FORCE SYSTEM.
2) NO REACTION EXISTS SUCH THAT CASE 1 IS POSSIBLE.

Ex. CASE 1:

(AE = Cons.)

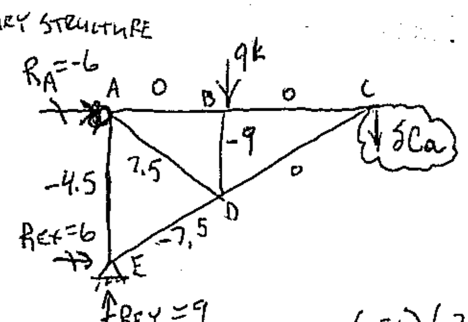


$B + R > 2J$
$7 + 4 > 2(5)$
$11 > 10$
INDETERMINATE, 1ST DEGREE

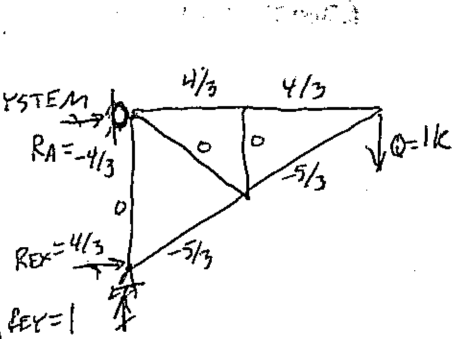
CREATE RELEASED STRUCTURE BY REMOVING ONE SUPPORT → CHOOSE R_C (NOTE: PINS & FIXED-ENDS

a.) PRIMARY STRUCTURE

P-SYSTEM



Q-SYSTEM



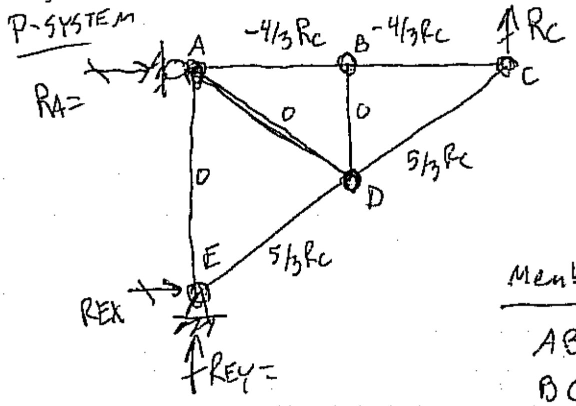
virtual work: $(1^k)\,\delta C_a = \dfrac{(-5/3)(-7.5)(25)(12)}{AE}$

$\delta C_a = \dfrac{3750}{AE}(\downarrow)$

... EXAMPLE, TRUSS CASE 1 CONT

b.) SECONDARY (Redundant load) Structure

P-SYSTEM



Q-SYSTEM
see primary structure

Member	F_P	F_Q	L	$F_P F_Q L$
AB	$-(4/3)R_c$	4/3	20'	$-(320/9)R_c$
BC	$-(4/3)R_c$	4/3	20'	$-(320/9)R_c$
CD	$(5/3)R_c$	-5/3	25'	$-(625/9)R_c$
DE	$(5/3)R_c$	-5/3	25'	$-(625/9)R_c$
AE	0	0	30'	0
AD	0	0	25'	0
BD	0	0	15'	0

$$(1k)\,\delta C_b = 2\left(\frac{-320}{9AE}\right) + 2\left(\frac{-625}{9AE}\right)$$

$$\delta C_b = -\frac{2520}{AE} R_c (\uparrow)$$

IN REALITY, $\Delta C = 0 \rightarrow \delta C_a + \delta C_b = 0$

$$\frac{3750}{\cancel{AE}} - \frac{2520}{\cancel{AE}} R_c = 0$$

$$R_c = \frac{3750}{2520} = 1.49\,k\,(\uparrow)$$

USE STATICS TO FIND R_A, R_{EX}, R_{EY}

$\Sigma F_y = 0]\; R_{EY} - 9 + 1.49 = 0$

$R_{EY} = 7.51\,k$

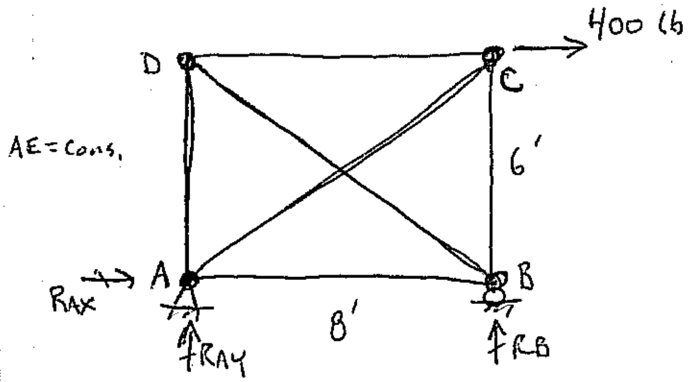
$\Sigma F_x = 0]\; R_A + R_{EX} = 0$

$\circlearrowleft \Sigma M_A = 0]\; -9(20) + 1.49(40) + R_{EX}(30) = 0$

$R_{EX} = 4.01\,k\,(\rightarrow)$

$R_A = -4.01\,k\,(\leftarrow)$

EXAMPLE, CASE 2



$B + R > 2J$

$6 + 3 > 2(4)$

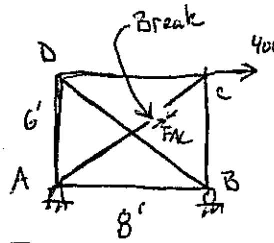
$9 > 8$

INDETERMINATE, 1ST DEGREE

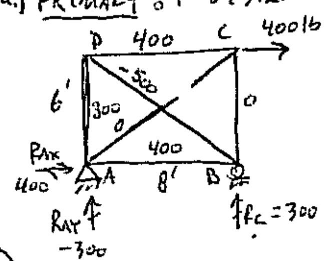
- WE CANNOT CREATE A STABLE, DETERMINATE STRUCTURE BY RELEASING ON OF THE SUPPORTS:
  1) $R_{AX} \rightarrow$ PARALLEL FORCE SYSTEM
  2) R_{AY} OR $R_B \rightarrow$ CONCURRENT FORCE SYSTEM

INSTEAD, WE INTRODUCE AN INTERNAL RELEASE:

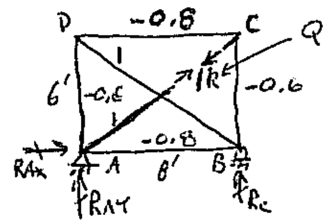
RELEASED STRUCTURE:



a) PRIMARY: P-SYSTEM



Q SYSTEM



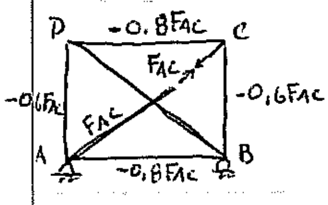
INTERNAL RELEASES EXPOSE 2 EQUAL AND OPPOSITE FORCES.

MEMBER	F_P	F_Q	L	$F_P F_Q L$
AB	400	-0.8	8	-2560
~~BC~~	~~0~~	~~-0.6~~	~~6~~	
CD	400	-0.8	8	-2560
AD	300	-0.6	6	-1080
CUT $\rightarrow$ ~~AC~~	~~0~~	~~+1~~	~~10~~	
BD	-500	+1	10	-5000
				$\Sigma = -11200\ k^2 ft$

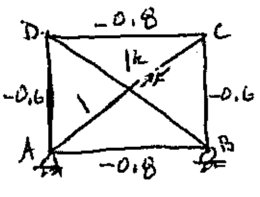
$1 \cdot \delta_{AC_a} = \frac{-11200}{AE}$

b.) RELEASED FORCE

P-SYSTEM



Q-SYSTEM



MEMBER	F_P	F_Q	L	$F_P F_Q L$
AB	-0.8 FAC	-0.8	8	5.12 FAC
BC	-0.6 FAC	-0.6	6	2.16 FAC
CD	-0.8 FAC	-0.8	8	5.12 FAC
AD	-0.6 FAC	-0.6	6	2.16 FAC
CUT $\rightarrow$ AC	+ FAC	+1	10	10 FAC
BD	+ FAC	+1	10	10 FAC
				$\Sigma = 34.56$

$1 \cdot \delta_{AC_b} = \frac{34.56 F_{AC}}{AE}$

Set to true displacement:

$\therefore \delta_{AC_a} + \delta_{AC_b} = 0$

$-\frac{11200}{1 \cancel{AE}} + \frac{34.56 F_{AC}}{1 \cancel{AE}} = 0$

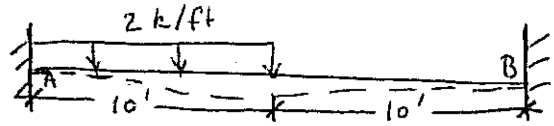
$\boxed{F_{AC} = 324\ lb\ (T)}$

Solve Remaining Bar Forces BY Method of Joints.

B.) MULTIPLE REDUNDANTS

- MULTIPLE REDUNDANTS ARE HANDLED IN A SIMILAR MANNER AS A SINGLE REDUNDANT:
  - KEY DIFFERENCES:
    1) RELEASED STRUCTURE STILL STABLE-DETERMINATE, SO NOW MULTIPLE RELEASES ARE REQUIRED TO GENERATE IT.
    2) NOW WE MUST TRACK DEFORMATIONS FOR EACH REDUNDANT.
    3) WE ALSO GENERATE MULTIPLE EQUATIONS OF COMPATIBILITY (ONE FOR EACH REDUNDANT)

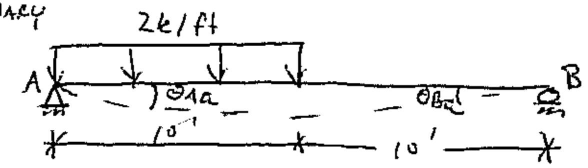
EXAMPLE: EI IS CONSTANT, IGNORE AXIAL EFFECTS.



2 REDUNDANTS: USE M_A & M_B

RELEASED STRUCTURE

a.) PRIMARY

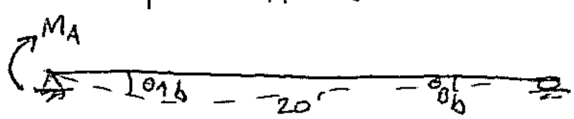


2 EQNS OF COMPATIBILITY

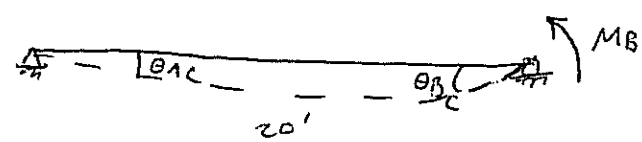
① $0 = \theta_{Aa} + \theta_{Ab} + \theta_{Ac}$

② $0 = \theta_{Ba} + \theta_{Bb} + \theta_{Bc}$

b.) REDUNDANT 1, M_A APPLIED



c.) REDUNDANT 2, M_B APPLIED



→ We Need to solve for $\theta_{Aa}, \theta_{Ba}, \theta_{Ab}, \theta_{Bb}, \theta_{Ac}$, & θ_{Bc}

COULD USE DOUBLE INTEGRATION OR VIRTUAL WORK:

- COULD ALSO USE TABLES: SEE HANDOUT

$$\theta_{Aa} = \frac{3wL^3}{128EI} = \frac{3(2)(20)^3}{128EI} = \frac{375}{EI} \qquad \theta_{Ba} = \frac{7wL^3}{384EI} = \frac{7(2)(20)^3}{384EI} = \frac{291.7}{EI}$$

$$\theta_{Ab} = \frac{M_A L}{3EI} = \frac{M_A(20)}{3EI} = \frac{6.67M_A}{EI} \qquad \theta_{Bb} = \frac{M_A L}{6EI} = \frac{M_A(20)}{6EI} = \frac{3.33M_A}{EI}$$

$$\theta_{Ac} = \frac{M_B L}{6EI} = \frac{3.33M_B}{EI} \qquad \theta_{Bc} = \frac{M_B L}{3EI} = \frac{6.67M_B}{EI}$$

CAREFUL ABOUT SIGN CONVENTION !!!

...EXAMPLE CONT.

APPLY EQNS. OF COMPATIBILITY:

① ROTATION AT A IS ZERO (FIXED-END)

$$0 = \theta_{Aa} + \theta_{Ab} + \theta_{Ac}$$

② ROTATION AT B IS ZERO (FIXED-END)

$$0 = \theta_{Ba} + \theta_{Bb} + \theta_{Bc}$$

$$① \quad 0 = \frac{375}{\cancel{EI}} + \frac{6.67 M_A}{\cancel{EI}} + \frac{3.33 M_B}{\cancel{EI}}$$

$$② \quad 0 = \frac{291.7}{\cancel{EI}} + \frac{3.33 M_A}{\cancel{EI}} + \frac{6.67 M_B}{\cancel{EI}}$$

2 SIMULTANEOUS EQNS, SOLVE BY FAVORITE METHOD
- → EQN SCALING / ADDITION
- → MATRIX METHOD, RREF (SAME THING)
- → SUBSTITUTION

$M_A = -45.8$ k·ft

$M_B = -20.8$ k·ft

MINUS SIGNS MEAN WHAT?

THROUGH STATICS WE GET: $R_{AY} = 16.25^k \uparrow$

$R_{BY} = 3.75^k \uparrow$