Lesson 22: Flexibility Method Continued

This lesson on Flexibility Method For Trusses
* Multiple Redundants

A) Application to Trusses:

→ Follow the same process as used for Beams & Frames --- Usually.

Determined for trusses: \( B + R > 2J \):
- Indeterminate Degree = \( 2J - (B + R) \)

\[ B = \# \text{Bars} \]
\[ R = \# \text{Reaction} \]
\[ J = \# \text{Joints} \]

(Notes concerning parallel and concurrent force systems apply)

→ 2 Cases for trusses
1) You are free to remove one reaction without creating a parallel or concurrent force system.
2) No reaction exists such that Case 1 is possible.

Example Case 1:

\( B + R > 2J \)
\( 7 + 4 > 2 \times 5 \)
\( 11 > 10 \)
Indeterminate, 1st Degree

Create released structure by removing one support → Choose \( R_C \) (Note: Pins vs. Fixed-Ends

a) Primary Structure

P-System

Virtual work: \( V^s \)

\[ V^s = \frac{(-5/3)(-7.5)(25/12)}{AE} \]

\[ S_C = \frac{3750}{AE} \]
EXAMPLE 1 TRUSS CASE I CONT

b) SECONDARY (Redundant load) Structure

\[
\begin{align*}
\text{P-System:} & \quad \text{Q-System:} \\
\text{P:} & \quad \text{Q:}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Member</th>
<th>F_p</th>
<th>F_q</th>
<th>L</th>
<th>F_p F_q L</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>(-\frac{1}{3}R_c)</td>
<td>(\frac{4}{3})</td>
<td>20'</td>
<td>(-\frac{320}{9}R_c)</td>
</tr>
<tr>
<td>BC</td>
<td>(-\frac{1}{3}R_c)</td>
<td>(\frac{4}{3})</td>
<td>20'</td>
<td>(-\frac{320}{9}R_c)</td>
</tr>
<tr>
<td>CD</td>
<td>(\frac{5}{3}R_c)</td>
<td>(-\frac{5}{3})</td>
<td>25'</td>
<td>(-\frac{625}{9}R_c)</td>
</tr>
<tr>
<td>DE</td>
<td>(\frac{5}{3}R_c)</td>
<td>(-\frac{5}{3})</td>
<td>25'</td>
<td>(-\frac{625}{9}R_c)</td>
</tr>
<tr>
<td>AE</td>
<td>0</td>
<td>0</td>
<td>25'</td>
<td>0</td>
</tr>
<tr>
<td>AO</td>
<td>0</td>
<td>0</td>
<td>25'</td>
<td>0</td>
</tr>
<tr>
<td>BD</td>
<td>0</td>
<td>0</td>
<td>15'</td>
<td>0</td>
</tr>
</tbody>
</table>

\(1k\) \(S_{C_b} = 2\left(-\frac{320}{9AE}\right) + 2\left(-\frac{625}{9AE}\right)\)

\[
S_{C_b} = -\frac{320R_c}{AE}
\]

IN REALITY \(\Delta C = 0 \Rightarrow S_{C_a} + S_{C_b} = 0\)

\[
\frac{3750}{AE} - \frac{2520}{AE} R_c = 0
\]

\[
R_c = \frac{3750}{2520} = 1.49 \text{ k}\ (\uparrow)
\]

USE STATICS TO FIND \(R_A, R_{EX}, \text{REY}\)

\[
\begin{align*}
\Sigma F_y = 0: & \quad \text{REY} - 9 + 1.49 = 0 \\
& \quad \text{REY} = 7.51 \text{ k}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x = 0: & \quad R_A + R_{EX} = 0 \\
\Sigma M_A = 0: & \quad -9(20) + 1.49(40) + R_{EX}(30) = 0 \\
& \quad R_{EX} = 4.01 \text{ k (\rightarrow)} \\
& \quad R_A = -4.01 \text{ k (\uparrow)}
\end{align*}
\]
EXAMPLE, CASE 2

\[ B + R > 2 \sqrt{S} \]
\[ G + 3 > 2 \] (4)
\[ 9 > 8 \]

INDETERMINATE 1ST DEGREE

- WE CANNOT CREATE A STABLE, DETERMINATE STRUCTURE
  BY RELEASING ON OF THE SUPPORTS.
  1) \( RA \rightarrow \) PARALLEL FORCE SYSTEM
  2) \( RA \) OR \( RB \) \( \rightarrow \) CONCURRENT FORCE SYSTEM

INSTEAD, WE INTRODUCE AN INTERNAL RELEASE.

RELEASED STRUCTURE

INTERNAL RELEASES EXPOSE 2 EQUAL AND OPPOSITE FORCES.

b) RELEASED FORCE

P-SYSTEM

\[ \begin{array}{c|c|c|c|c}
\text{MEMBER} & F_p & F_q & L & F_p F_q L \\
\hline
AB & -0.8 & 400 & 8 & -2560 \\
BC & 0 & -0.6 & 6 & 0 \\
CD & 0 & 0.8 & 8 & 2560 \\
AD & 0 & -0.6 & 6 & -1080 \\
AC & 0 & +1 & 10 & 10 \\
BD & -0.8 & +1 & 10 & -5000 \\
\end{array} \]

\[ \varepsilon = \frac{-1200}{AE} \]

\[ \frac{F_A C}{AE} = \frac{34.56}{AE} \]

16 \( \varepsilon \) \( AB \) = \( \frac{34.56}{AE} \)

9 \( \varepsilon \) \( AB \) = \( \text{true displacement} \)

2. \( \varepsilon \) \( AB \) + \( \varepsilon \) \( AB \) = 0

\[ \varepsilon = \frac{34.56}{AE} \]

FAC = 324 lb (T)

SOLVE REMAINING BAR FORCES BY METHOD OF JOINTS.
B) MULTIPLE REDUNDANTS

- MULTIPLE REDUNDANTS ARE HANDLED IN A SIMILAR MANNER AS A SINGLE REDUNDANT
- KEY DIFFERENCES?
  1) RELEASED STRUCTURE STILL STABLE - DETERMINATE / SO NOW MULTIPLE RELEASES ARE REQUIRED TO GENERATE IT.
  2) NOW WE MUST TRACK DEFORMATIONS FOR EACH REDUNDANT.
  3) WE ALSO GENERATE MULTIPLE EQUATIONS OF COMPATIBILITY (ONE FOR EACH REDUNDANT).

EXAMPLE:

$E_I$ is constant, ignore axial effects.

**RELEASED STRUCTURE**

a) Primary:

[Diagram showing a structure with primary loads and reactions]

2 REDUNDANTS:

USE $MA$ & $MB$

b) Redundant 1, $MA$ Applied

[Diagram showing reactions and displacements due to $MA$]

2 EQNS OF COMPATIBILITY

1) $O = \theta_{A} + \theta_{AB} + \theta_{AC}$
2) $O = \theta_{B} + \theta_{BB} + \theta_{BC}$

c) Redundant 2, $MB$ Applied

[Diagram showing reactions and displacements due to $MB$]

--- We need to solve for $\theta_{AC}$, $\theta_{BA}$, $\theta_{AB}$, $\theta_{BB}$, $\theta_{AC}$, $\theta_{BC}$
Could use double integration or virtual work;
Could also use tables, see handout

$\theta_{A} = \frac{3wL^3}{128EI} = \frac{3(2)(2)^3}{128EI} = 375$:

$\theta_{B} = \frac{7wL^3}{384EI} = \frac{7(2)(2)^3}{384EI} = 291.7$:

$\theta_{AB} = \frac{MAL}{3EI} = \frac{Ma(2)}{3EI} = 6.67Ma$:

$\theta_{BB} = \frac{MAL}{6EI} = \frac{Ma(2)}{6EI} = 3.33Ma$:

$\theta_{AC} = \frac{MBL}{6EI} = \frac{3.33MB}{6EI}$:

$\theta_{BC} = \frac{MBL}{3EI} = \frac{6.67MB}{6EI}$:

CAREFUL ABOUT SIGN CONVENTION!!!
EXAMPLE CONT.

Apply eqns. of compatibility

1. Rotation at A is zero (fixed-end)
   \[ \theta_{Aa} + \theta_{Ab} + \theta_{Ac} = 0 \]

2. Rotation at B is zero (fixed-end)
   \[ \theta_{Ba} + \theta_{Bb} + \theta_{Bc} = 0 \]

\[ \begin{align*}
   \theta_{Aa} &= \frac{375}{EI} + 6.67 \frac{MA}{EI} + 3.33 \frac{MB}{EI} \\
   \theta_{Ba} &= \frac{291.7}{EI} + 3.33 \frac{MA}{EI} + 6.67 \frac{MB}{EI}
\end{align*} \]

2 simultaneous eqns. solve by favorite method

- Eqn. scaling/addition
- Matrix method, REF (same thing)
- Substitution

\[ \begin{align*}
   MA &= -45.8 \text{ k.lf} \\
   MB &= -20.8 \text{ k.lf}
\end{align*} \]

Through statics we get:

\[ \begin{align*}
   R_{AY} &= 16.25 \text{ k} \\
   R_{BY} &= 3.75 \text{ k}
\end{align*} \]