LESSON 24 INTRO. TO DIRECT STIFFNESS METHOD

READING: TEXT Ch. 16.1 - 16.2

- INTRODUCE DIRECT STIFFNESS METHOD
- QUICK COMPARISON TO FLEXIBILITY METHOD
- SPRING ANALOGY

A.) INTRODUCTION TO DIRECT STIFFNESS METHOD;

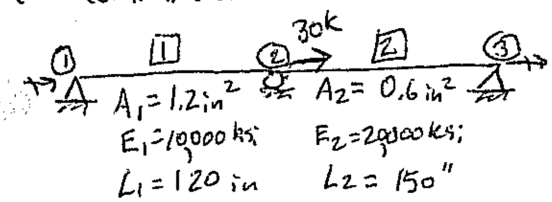
STEPS:

- FIND EQUILIBRIUM AT JOINTS IN TERMS OF UNKNOWN JOINT DEFLECTIONS
- SOLVE FOR DEFLECTIONS
- SOLVE FOR MEMBER FORCES

- ADVANTAGES: - ALGORITHMIC: CAN AUTOMATE
  - BASIS FOR COMPUTER-BASED ANALYSIS SOFTWARE PACKAGES
  - CAN EASILY HANDLE ANY GEOMETRY
  - MANY DEGREE INDETERMINACY OK
- DISADVANTAGES: MANY COMPUTATIONS / TEDIOUS

B.) QUICK COMPARISON TO FLEXIBILITY METHOD;

SIMPLE TRUSS:

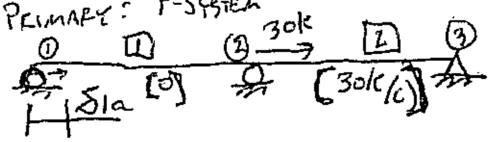


2 REACTIONS
− 1 EQN. OF EQUILIBRIUM
1 DEGREE INDETERMINATE

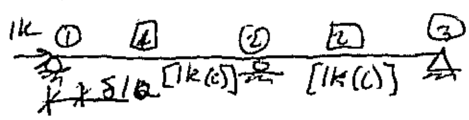
BY FLEXIBILITY METHOD:

Release joint ①:

PRIMARY: P-SYSTEM

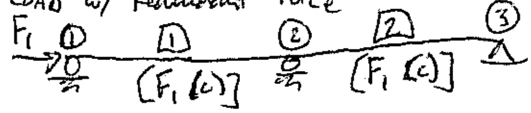


Q-SYSTEM



$$\delta_{1a} = \frac{F_{P_2} F_{Q_2} L_2}{A_2 E_2} = \frac{-30(-1)(150)}{(0.6)(20,000)} = 3/8 \text{ in}$$

P-System: LOAD w/ Redundant Force



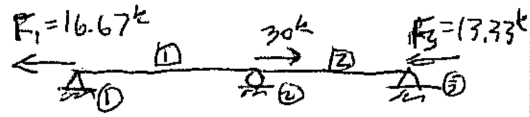
Q-SYSTEM (see above)

$$\delta_{1b} = \frac{F_{P_1} F_{Q_1} L_1}{A_1 E_1} + \frac{F_{P_2} F_{Q_2} L_2}{A_2 E_2} = \frac{(-F_1)(-1)\,120}{1.2(10,000)} + \frac{(-F_1)(-1)(150)}{0.6(20,000)} = 0.0225 F_1 \text{ (in)}$$

SIMPLE TRUSS EXAMPLE (FLEXIBILITY METHOD) CONT...

EQN. OF COMPATIBILITY:

$$0 = 3/8 + (0.0225) F_1$$



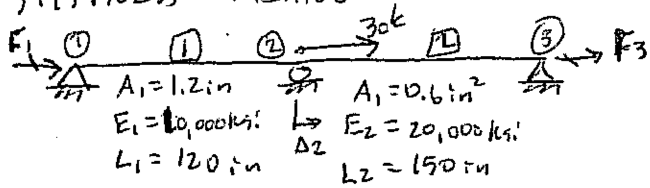
$$F_1 = -16.67 \ k \ (\leftarrow)$$

Now:

$\overset{+}{\rightarrow} \Sigma F_x = 0]$ $F_1 + F_3 + 30 = 0$

$$F_3 = -13.33 k \leftarrow$$

BY STIFFNESS METHOD

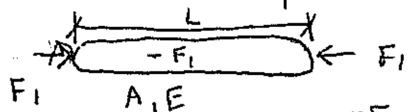


IMAGINE WHAT HAPPENS WHEN JOINT 2 DEFORMS

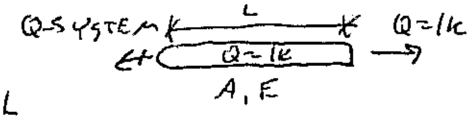
$$\Delta L_1 = \Delta 2 = \Delta L 2 \qquad \text{(COMPATIBILITY)}$$

FOR 1 BAR, WE CAN EXPRESS A RELATIONSHIP BETWEEN ΔL & END FORCE

P-SYSTEM



Q-SYSTEM



$$1 \cdot \Delta L = \frac{-F_1 (1) L}{AE}$$

$$\text{OR:} \quad F_1 = -\left(\frac{AE}{L}\right) \Delta L$$

FOR OUR EXAMPLE THEN, $F_1 = \Delta L_1 \dfrac{A_1 E_1}{L_1}$ & $F_2 = \Delta L_2 \dfrac{A_2 E_2}{L_2}$

SINCE $\Delta L_1 = \Delta L 2 = \Delta 2$:

$$F_1 = -\frac{1.2 (10,000)}{120} \Delta_2 = -100 \Delta 2$$

$$F_2 = -\frac{0.6 (20,000)}{150} \Delta_2 = -80 \Delta_2$$

USING STATIC EQUILIBRIUM:

$\overset{+}{\rightarrow} \Sigma F_x = 0]$ $30 + F_1 + F_3 = 0$

$$30 - 100 \Delta_2 - 80 \Delta_2 = 0$$

$$\Delta 2 = 1/6 \text{ in}$$

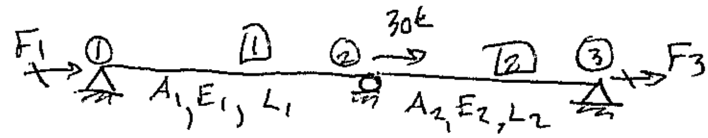
Plug in $\Delta 2$:

$$F_1 = 100 \Delta_2 = -100 (1/6) = -16.67 k$$

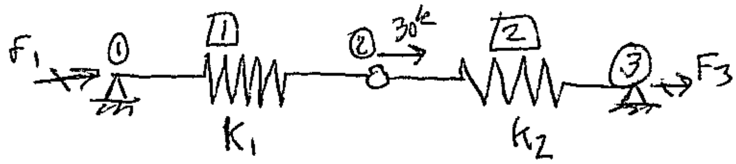
$$F_2 = 80 \Delta_2 = -80 (1/6) = -13.33 k$$

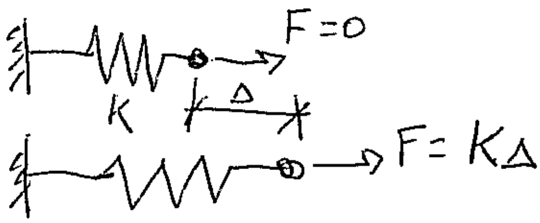
C.) SPRING ANALOGY

TRUSS



SPRING SYSTEM





FOR TRUSS ELEMENTS, $K = \frac{AE}{L}$

(FROM EXAMPLE, $F_1 = \frac{A_1 E_1}{L_1} \Delta L_1$ & $F_2 = \frac{A_2 E_2}{L_2} \Delta L_2$)

- TRUSS BARS ARE LIKE SPRINGS! (Related to axial deflection).
- WE WILL ALSO SEE, BEAMS WILL HAVE THEIR OWN SPRING CONSTANTS RELATED TO DISP. & ROTATION.