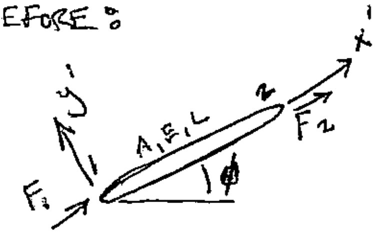
LESSON 28: INCLINED BARS AND COORDINATE TRANSFORM

TOPICS:

- INCLINED BARS
- TRANSFORMATION FROM LOCAL TO GLOBAL COORDINATES

A) INCLINED BARS

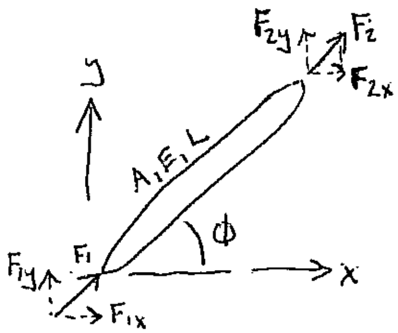
- FROM BEFORE:



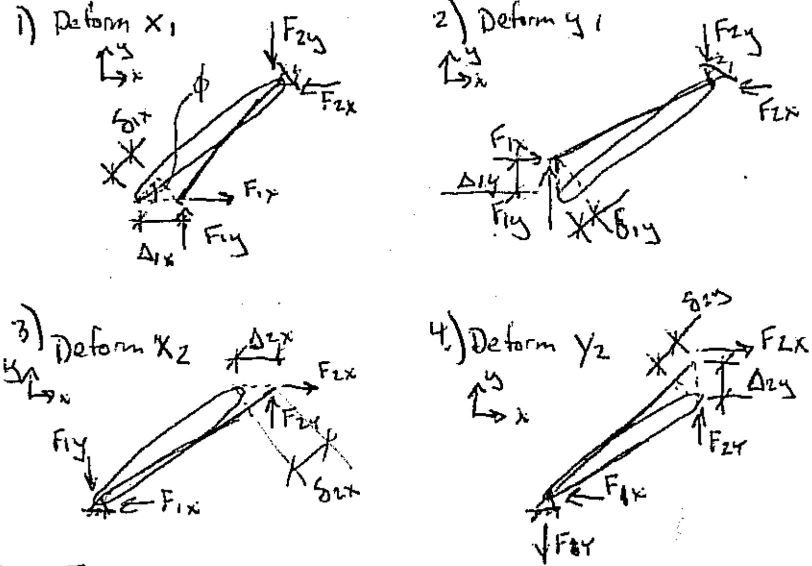
$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} AE/L & -AE/L \\ -AE/L & AE/L \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

CONVIENT BECAUSE TRUSS BAR FORCES ONLY ACT AXIALLY.

- INCLINED BARS DO NOT LINE UP WITH GLOBAL X & Y



$$F_1 = F_{1x} + F_{1y}$$
$$F_2 = F_{2x} + F_{2y}$$



__FORCES:__ $F_1 = \frac{AE}{L}\delta$, $F_2 = -F_1$

Case 1: $F_1 = \frac{AE}{L}\delta_{1x} = \frac{AE}{L}\left(\cos\phi\,\Delta_{1x}\right)$

$$F_{1x} = F_1 \cos\phi = \frac{AE}{L}(\cos^2\phi)\Delta_{1x}$$
$$F_{1y} = F_1 \sin\phi = \frac{AE}{L}(\cos\phi)(\sin\phi)\Delta_{1x}$$
$$F_{2x} = -F_{1x} = -\frac{AE}{L}(\cos^2\phi)\Delta_{1x}$$
$$F_{2y} = -F_{1y} = -\frac{AE}{L}(\cos\phi)(\sin\phi)\Delta_{1x}$$

Case 2: $F_1 = \frac{AE}{L}\delta_{1y} = \frac{AE}{L}(\sin\phi)\Delta_{1y}$

$$F_{1x} = \frac{AE}{L}\sin\phi\cos\phi\,\Delta_{1y}$$
$$F_{1y} = \frac{AE}{L}\sin^2\phi\,\Delta_{1y}$$
$$F_{2x} = -\frac{AE}{L}\sin\phi\cos\phi\,\Delta_{1y}$$
$$F_{2y} = \frac{-AE}{L}\sin^2\phi\,\Delta_{1y}$$

Case 3:

$$F_1 = \frac{AE}{L}\delta_{2x} = \frac{AE}{L}\cos\phi\,\Delta_{2x}$$

$$F_{1x} = -\frac{AE}{L}\cos^2\phi\,\Delta_{2x}$$
$$F_{1y} = -\frac{AE}{L}\sin\phi\cos\phi\,\Delta_{2x}$$
$$F_{2x} = AE/L\,\cos^2\phi\,\Delta_{2x}$$
$$F_{2y} = AE/L\,\sin\phi\cos\phi\,\Delta_{2x}$$

Case 4:

$$F_1 = \frac{AE}{L}\delta_{2y} = \frac{AE}{L}\sin\phi\,\Delta_{2x}$$

$$F_{1x} = -AE/L\,\sin\phi\cos\phi\,\Delta_{2y}$$
$$F_{1y} = -AE/L\,\sin^2\phi\,\Delta_{2y}$$
$$F_{2x} = AE/L\,\sin\phi\cos\phi\,\Delta_{2y}$$
$$F_{2y} = AE/L\,\sin^2\phi\,\Delta_{2y}$$

$$s = \sin\phi\ ,\quad c = \cos\phi$$

$$Q_{1x} = \Sigma F_{1x} = AE/L\left[c^2\Delta_{1x} + sc\,\Delta_{1y} - c^2\Delta_{2x} - sc\,\Delta_{2y}\right]$$
$$Q_{1y} = \Sigma F_{1y} = AE/L\left[sc\,\Delta_{1x} + s^2\Delta_{1y} - sc\,\Delta_{2x} - s^2\Delta_{2y}\right]$$
$$Q_{2x} = \Sigma F_{2x} = AE/L\left[-c^2\Delta_{1x} - sc\,\Delta_{1y} + c^2\Delta_{2x} + sc\,\Delta_{2y}\right]$$
$$Q_{2y} = \Sigma F_{2y} = AE/L\left[-sc\,\Delta_{1x} - s^2\Delta_{1y} + sc\,\Delta_{2x} + s^2\Delta_{2y}\right]$$

GLOBAL COORDINATES $\Downarrow$ MATRIX FORM

$$\begin{bmatrix} Q_{1x} \\ Q_{1y} \\ Q_{2x} \\ Q_{2y} \end{bmatrix} = \frac{AE}{L} \begin{array}{c} \begin{array}{cccc} 1x & 1y & 2x & 2y \end{array} \\ \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \end{array} \begin{array}{c} \\ 1x \\ 1y \\ 2x \\ 2y \end{array} \begin{bmatrix} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{2x} \\ \Delta_{2y} \end{bmatrix}$$

$$\bar{q}_{4\times1} = \bar{K}_{4\times4}\ \bar{\delta}_{4\times1}$$

CONVERSION BETWEEN LOCAL DEFORMATION, δ, & global deflection, Δ:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} \Delta_{1x} \\ \Delta_{1y} \\ \Delta_{2x} \\ \Delta_{2y} \end{bmatrix}$$

$$\bar{\delta}_{2\times1} = \bar{T}\ \bar{\delta}_{4\times1}$$

See EXAMPLES 17.2 & 17.3 & 17.4

B.) TRANSFORMATION MATRIX

$$\bar{K}_{local}\Big|_{4\times4} = \bar{T}^T \, \bar{K}_{local_{2\times2}} \, \bar{T}$$

$$\bar{T} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}_{2\times4}, \quad \bar{T}^T = \text{transpose}(\bar{T}) = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix}_{4\times2}$$

- EXAMPLE OF A MATRIX AS A LINEAR TRANSFORMATION. (CORE OF LINEAR ALGEBRA).