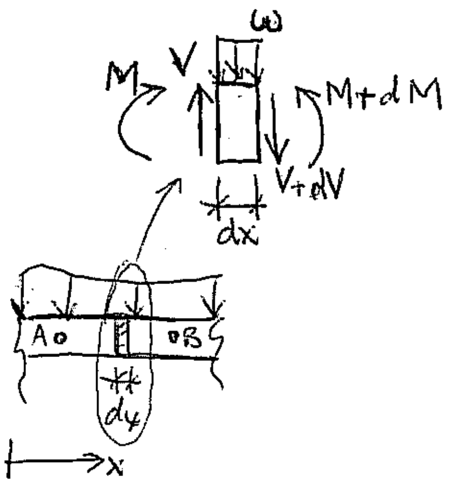
LESSON 7: Shear AND MOMENT DIAGRAMS

READING: Text Ch. 5

→ IN BEAM DESIGN, WE WANT TO FIND MAXIMUM SHEARS AND MOMENTS

→ SHEAR AND MOMENT DIAGRAMS PROVIDE THIS INFORMATION GRAPHICALLY

A. RELATIONSHIPS BETWEEN LOAD, SHEAR, & MOMENT:



Shear & Load:

$$dV = \omega\, dx$$

$$\Delta V_{A\to B} = V_B - V_A = \int_A^B dV = \int_A^B \omega\, dx$$

2 consequences:

1) $\Delta V_{A\to B}$ = area under load curve between A & B

2) $\dfrac{dV}{dx} = \omega$

Moment & Shear:

$$dM = V\, dx$$

$$\Delta M_{A\to B} = M_B - M_A = \int_A^B dM = \int_A^B V\, dx$$

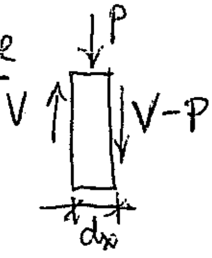
2 consequences:

1) $\Delta M_{A\to B}$ = area under shear curve between A & B

2) $\dfrac{dM}{dx} = V$

→ CONCENTRATED LOADS RESULT IN A JUMP:

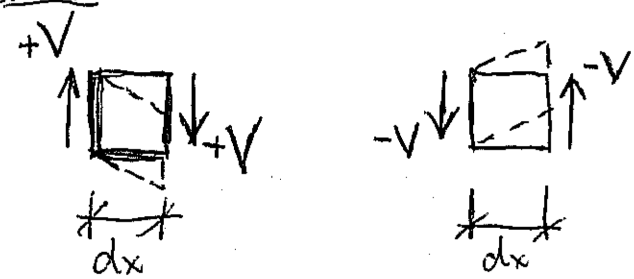
SHEAR



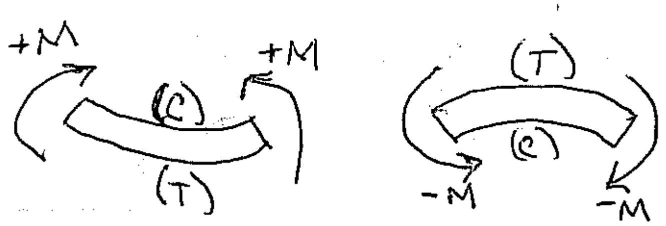
MOMENT

B. Sign Convention:

SHEAR



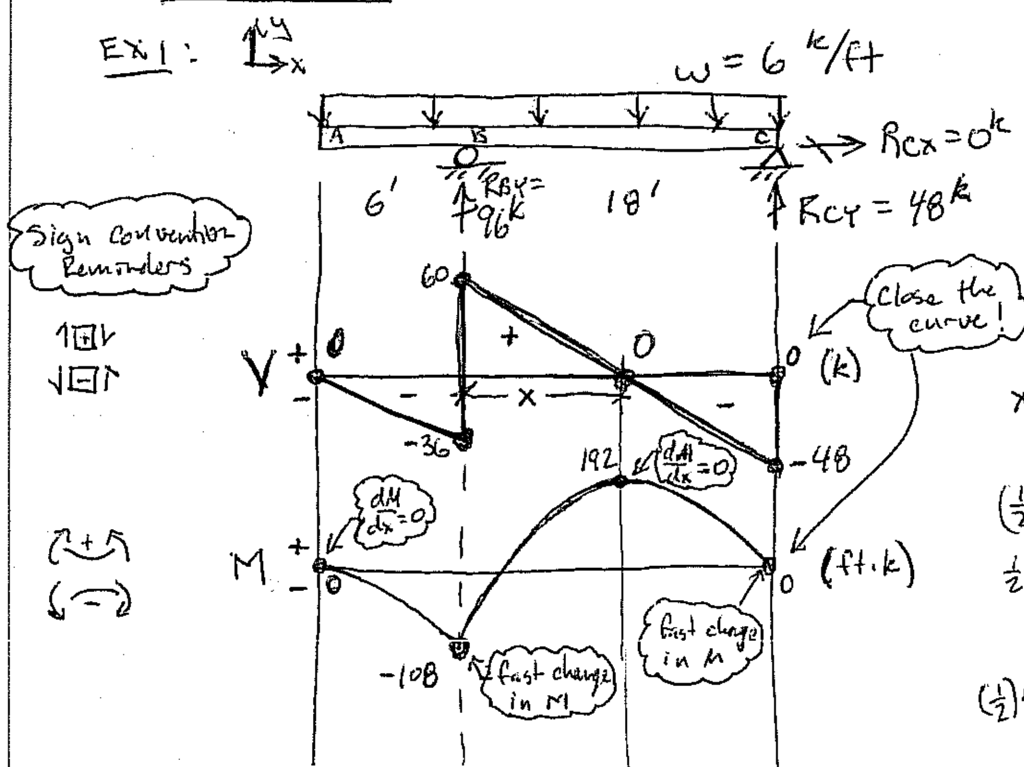
MOMENT



"SMILEY BEAM" "FROWNY BEAM"

C. Examples:

Ex 1:



$$\begin{array}{r} 18' \\ \times\ 6\ k/ft \\ \hline 108\ k \end{array}$$

$$x = \frac{60k}{6k/ft} = 10'$$

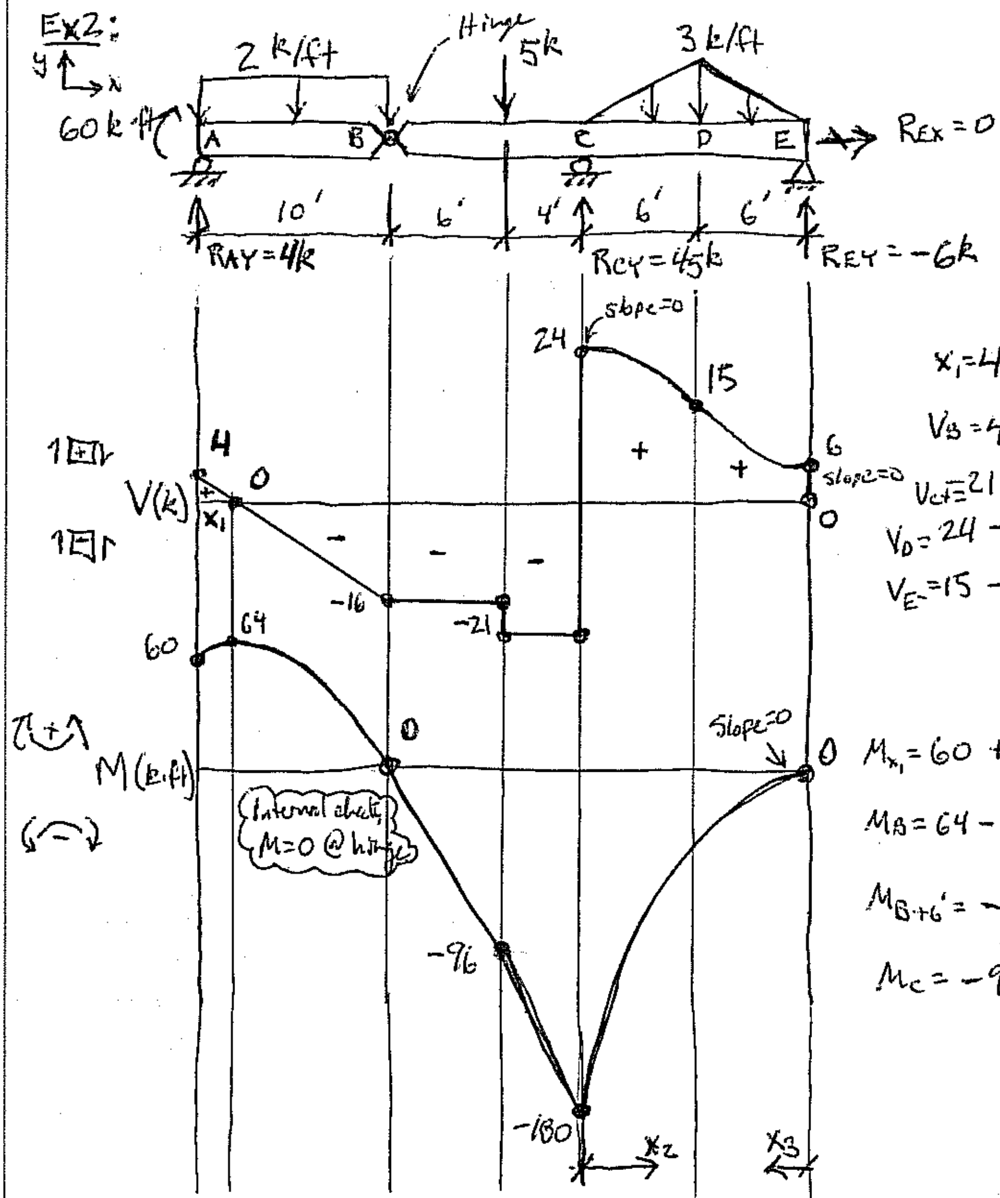
$(\frac{1}{2})36(6) = 108\,kft$

$\frac{1}{2}(60)(10) = 300\,kft$
$$\begin{array}{r} -108 \\ \hline 192 \end{array}$$

$(\frac{1}{2})48(8) = 192$
$$\begin{array}{r} -192 \\ \hline 0 \end{array}$$

→ Remember: Sign Convention
Units
Close the curve
Use Construction Lines

Ex2:



$x_1 = 4k / 2 k/ft = 2 ft$

$V_B = 4 - 2(10) = -16\,k$

$V_{C^+} = 21 + 45 = 24k$

$V_D = 24 - \frac{1}{2}(3)(6) = 24 - 9 = 15^k$

$V_{E^-} = 15 - \frac{1}{2}(3)(6) = 6^k$

$M_{x_1} = 60 + \frac{1}{2}(4)x_1 = 64\ k\cdot ft$

$M_B = 64 - 16(\frac{1}{2})(10-2) = 0\ k\cdot ft$

$M_{B+6'} = -16(6) = -96\ k\cdot ft$

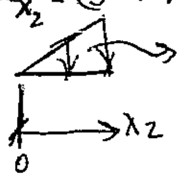
$M_C = -96 - 21(4) = -180\ k\cdot ft$

M_E should $= 0\ k\cdot ft$, does it?

$$M_E = M_C + \int_C^E V(x)\,dx$$

$$= -180^{k\cdot ft} + \int_{0'}^{6'} 24^k - \frac{1}{4}x_2^2 (k/ft^2)\,dx_2 + \int_{0'}^{6'} 6^k + \frac{1}{4}x_3^2 (k/ft^2)\,dx_3$$

$\omega_{x_2} = (3k/ft / 6ft)\,x_2 = \frac{1}{2}x_2 (k/ft^2)$ | $\omega_{x_3} = \frac{1}{2}x_3 (k/ft^2)$



$\text{Area} = \frac{1}{2}(\omega_{x_2})(x_2)$

$= \frac{1}{2}\left(\frac{1}{2}\frac{k}{ft^2}x_2\right)x_2$

$= \frac{1}{4}x_2^2\ (k/ft^2)$

$\text{Area} = \frac{1}{4}x_3^2\ (k/ft^2)$

$$= -180^{k\cdot ft} + \left[24^{(k)}x_2 - \frac{1}{12}x_2^3 (k/ft^2)\right]\Big|_{0'}^{6'} + \left[6^k x_3 + \frac{1}{12}x_3^3 (k/ft^2)\right]\Big|_{0'}^{6'}$$

$$= -180k\cdot ft + (24k)6' - \frac{1}{12}(6')^3 (k/ft^2) - 0 + (6k)(6') + \frac{1}{12}(6')^3 (k/ft^2) - 0$$

$$= -180k\cdot ft + 126\ k\cdot ft + 54\ k\cdot ft = 0\ k\cdot ft$$