- Sketching Deflected Shapes: Inflection Points

At points where the moment curve is zero, there exists a point at which direction of curvature changes, Inflection Point.

Example:

![Diagram of deflected shapes with inflection point]

Lesson 9:

Principle of Superposition:

For structures that behave in a linear-elastic fashion, forces & displacements due to a set of loads are equal to the sum of the forces and displacements due to each load individually.
**Superposition Illustration**

Equivalent to:

\[
\begin{align*}
20 \text{kN} & \cdot \text{m} \\
5 \text{kN/m} & \downarrow \\
12 \text{m} & \\
\end{align*}
\]

\[
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20 \text{kN} & \cdot \text{m} \\
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12 \text{m} \\
\end{align*}
\]

\[
\begin{align*}
20 \text{kN} & \cdot \text{m} \\
5 \text{kN/m} & \downarrow \\
12 \text{m} & \\
\end{align*}
\]

Useful? Think about this beam:

\[
\begin{align*}
20 \text{kN} & \cdot \text{m} \\
5 \text{kN/m} & \downarrow \\
12 \text{m} \\
\end{align*}
\]

Limitations on Superposition:

- Linear systems only:
  - Not linear if deflections are large

\[
\begin{align*}
P_x & \rightarrow A_1 \\
\rightarrow \\
\rightarrow \\
\end{align*}
\]

\[
\begin{align*}
P_y & \rightarrow A_2 \\
\rightarrow \\
\rightarrow \\
\end{align*}
\]

\[
\begin{align*}
P_x & \rightarrow A_1 + A_2 \\
\rightarrow \\
\rightarrow \\
\end{align*}
\]

If \( \Delta \) is "large":

- Elastic systems only:
  - Not elastic if yield strain (stress) has been exceeded.

Material non-linearity:

Breaking \( \sigma_b > \sigma_y \rightarrow \) Inelastic. No superposition.
Superposition and Deflection:
- Deflected shape due to 2 loads:
  \[ \bar{A} \quad \sqrt{P_2} \]
  ARBITRARY POINT, A
- Can be found by adding deflected shape from each load acting alone.

\[ \Delta A = \Delta A_{P_1} + \Delta A_{P_2} \]

More
DIRECT COMPUTATION OF MOMENT DIAGRAM w/ SUPERPOSITION

**Example**

- 150 k-ft
- \[ \frac{1}{2} (5)(15) = 37.5 k \]

Replace supports with fixed-end at right but keep reaction forces.

150 k-ft equals

\[ M_t \]

plus

\[ M_2 \]

\[ M_2 = \frac{1}{2} (15)(15) = 187.5 \]

Ends Moment = -150 + 377.5 - 187.5 = 0

Why did this work?