Lesson 11  Deflection: Moment-Area Method

**Reading: Text 9.3**

1) The change in slope between two points on a smooth continuous curve is equal to the area under the \(\frac{M}{EI}\) curve.

(Note: Smooth implies no hinges)

2) The tangential deviation at a point on a smooth continuous curve from a tangent line drawn at a second point on that curve is equal to the moment about the first point of the area under the \(\frac{M}{EI}\) curve.

Limits: Shallow Curves
Elastic Behavior
Smooth Curves (No hinges)
How To Apply Moment-Area Method

- Draw Moment Diagram (Shear diagram is helpful)
- Sketch the deflected shape
- Identify points of known slope:

Example: Cantilever Beam

\[ t_{BA} = \int_{A}^{B} \frac{Mx}{EI} \, dx \]

- The slope of a beam at a fixed-end support is zero, i.e., the tangent line is horizontal.

If we had an expression for \( t_{BA} \), we could know the deflection at B.

- Moment-Area Method:

\[ M = P \times ax \]

\[ \int P \, dx = \int \frac{Mx}{EI} \, dx \]

\[ \left[ \frac{1}{EI} \left( \frac{1}{2} P x^2 \right) \right]_0^L = \frac{PL^3}{3EI} \]

- Imagine Moment curve is \( \frac{M}{EI} \)

Distributed load, what is its "moment" around B

\[ \text{Area} = \frac{1}{2} \left( \frac{PL}{EI} \right) (L) \]

\[ \text{Moment arm} = \frac{2L}{3} \]

- \( \Delta = t_{BA} = \text{"Moment"} = \left( \frac{PL^2}{EI} \right) \left( \frac{2L}{3} \right) \)

\[ \Delta = -\frac{PL^3}{3EI} \]

Can solve graphically, saves time, increase accuracy (maybe)
**Example of Symmetric Members**

Here, slope at D = 0

**Find** \( \Delta_D \) and \( \Delta_c \) and \( \Theta_c \)

\( E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2 \)

\( I = 6 \times 10^6 \text{ mm}^4 = 6 \times 10^{-6} \text{ m}^4 \)

\[ \Theta_c = -\Delta \Theta_{CD} = \text{Area of}: \]

\[ -\Delta \Theta_{CD} = 30(3) + \frac{1}{2}(30)(3) \]

\[ = 135 \text{ kNm}^2 \]

\[ \frac{EI}{E} \]

\[ \Theta_c = 0.113 \text{ (rad.)} \]

\[ \Delta_D = -t_{BD} \]

\[ t_{BD} = \int_D^B \frac{M}{EI} dx \]

**Use Graphical Method**

\[ t_{BD} = \frac{1}{2}(60)(6)(4)(\frac{1}{EI}) \]

\[ = \frac{720}{EI} \text{ (kNm)} \]

\[ -\Delta_D = t_{BD} = 0.6 \text{ m} \]

\[ \Delta_D = -0.6 \text{ m} \]

\[ \Delta_c = -t_{CD} - t_{CO} \]

\[ t_{CD} = \frac{1}{2}(30)(3)(3) \]

\[ = 135 \text{ kNm}^3 \]

\[ \frac{EI}{E} \]

\[ t_{CO} = 1.875 \text{ m} \]

\[ m = 0.19 \text{ m} \]

\[ \Delta c = -0.6 - 0.19 \]

\[ \Delta c = -0.79 \text{ m} \]
Moment-Area Method: Things to Keep in Mind

1. Note sign of \( \frac{M}{EI} \) Curvature

<table>
<thead>
<tr>
<th>( \frac{M}{EI} )</th>
<th>Slope</th>
<th>Tangential Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) to ( B )</td>
<td>( \Delta \theta_{AB} ) positive</td>
<td>TAB pos, TBA pos.</td>
</tr>
<tr>
<td>( A ) to ( B )</td>
<td>( \Delta \theta_{BA} ) negative</td>
<td>TAB neg, TBA neg.</td>
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2. If \( I \) varies in beam \( \frac{M}{EI} \) curve is affected.

Ex.

Also see Example 9.4

3. Direction of "moment arm"
   For TAB, find "moment" around A.
   For TBA, find "moment" around B.

4. Sometimes multiple applications are necessary to get deflection of interest.

5. What is the difference between a definite and indefinite integral? Why am I asking you this?