Problem 1.) Double Integration Method

a.) For the \( x_1, x_2 \) coordinate system shown on the beam below, find the slope and deflection equations, \( EI \) is constant. Expressions for moment (\( M_1, M_2 \)) are given. (15 Points)

\[
\begin{align*}
0 \leq x_1 \leq 2L & \quad M_1(x) = -\frac{P}{2} x_1 \\
& \quad EI \frac{dy_1}{dx} = \frac{P}{4} x_1^2 + C_1 \\
& \quad EI y_1 = -\frac{P}{12} x_1^3 + C_1 x_1 + C_2 \\
2L \leq x_2 \leq 3L & \quad M_2(x_2) = P x_2 - P L \\
& \quad EI \frac{dy_2}{dx_2} = \frac{P}{2} x_2^2 - P L x_2 + C_3 \\
& \quad EI y_2 = \frac{P}{6} x_2^3 - \frac{P L}{2} x_2^2 + C_3 x_2 + C_4
\end{align*}
\]

b.) Indicate 4 appropriate boundary conditions you might use to solve for your constants of integration from part a.). Set up the solutions for your constants based on the boundary conditions you have selected but do not solve for the constants. (15 Points)

1) \( y_1(0) = 0 \); \( EI(0) = -\frac{P}{12} (0)^3 + C_1(0) + C_2 \)

2) \( y_1(2L) = 0 \); \( EI(0) = -\frac{P}{12} (2L)^3 + C_1(2L) + C_2 \)

3) \( y_2(2L) = 0 \); \( EI(0) = \frac{P}{6} (2L)^3 - \frac{P L}{2} (2L)^2 + C_3 (2L) + C_4 \)

4) \( \frac{dy_1}{dx_1}(2L) = \frac{dy_2}{dx_2}(2L) \); \( -\frac{P}{4} (2L)^2 + C_1 = \frac{P}{2} (2L)^2 - P L (2L) + C_3 \)
PROBLEM 2) CONSIDER THE FOLLOWING BEAM WITH CROSS-SECTIONAL PROPERTIES AND SHEAR & MOMENT DIAGRAMS SHOWN.

\[ 240k \text{ ft}^4 \]
\[ 2k \]
\[ 18k \]
\[ 10k \]
\[ 0 \]
\[ -2 \]
\[ -10 \]
\[ 240 \]

a.) Find the slope at B, use the moment-area method. (10 points)

b.) Find the displacement at C, use the moment-area method. (15 points)

NEXT PAGE
a.) Slope at $B = 0 + A_1$

$$\theta_B = -\frac{2200}{EI} \text{(k-ft)}$$

$$= 0 + \left[ \frac{1}{2} \left( \frac{20}{EI} \right) (20) + \left( \frac{100}{EI} \right) (20) \right]$$

b.) $\Delta_c = \frac{t_{CA}}{A_1} = \text{Moment} A_1 + \text{Moment} A_2$

$$= \left( \frac{1}{2} \left( \frac{20}{EI} \right) (20) \right) + \frac{10}{3} 20(20) + \frac{1}{2} \left( \frac{200}{EI} \right) (20) \left( \frac{2}{3} (20) \right)$$

$$\Delta_c = \left( \frac{6667}{EI} + \frac{60000}{EI} + \frac{26667}{EI} \right)$$

$$\Delta_c = \frac{93300}{EI} \text{(k-ft)}$$
Problem 3: Method of Virtual Work

a.) For the truss shown below, find the vertical displacement at node D. For all members, $E = 29,000$ kips, and $A = 5.0 \text{ in}^2$. Some potentially helpful internal loads have been given. Use virtual work. (20 points)

Structure under load:
(loads are in kips)

![Diagram of truss structure under load]

$\Delta y = ?$

Structure under some other load:

![Diagram of truss structure under some other load]

b.) Suppose that you wanted to eliminate this deflection (i.e., make $\Delta y = 0$). Also suppose you wanted to do so by heating up member FG. How much temperature change would you need in FG to make $\Delta y$ zero again? (5 points)
a) \[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Member} & \mathbf{F_Q (k)} & \mathbf{F_P (k)} & \mathbf{L (ft)} & \mathbf{F_P F_Q L (k^2 \cdot ft)} \\
\hline
\text{AB} & 1 & 5 & 10 & 50 \\
\text{BC} & 1.4 & 7.1 & 5 \sqrt{2} & 70.6 \\
\text{CD} & 1.4 & 0 & & \\
\text{DE} & 1 & 0 & & \\
\text{EE} & 1 & 0 & & \\
\text{FG} & -2 & -15 & 10 & 300 \\
\text{CE} & & 0 & & \\
\text{CF} & & 0 & & \\
\text{BF} & -1 & -5 & 10 & 50 \\
\text{AH} & 1.4 & 14 & 5 \sqrt{2} & 139 \\
\text{FH} & 1.4 & 14 & 5 \sqrt{2} & 139 \\
\text{BH} & & 0 & & \\
\hline
\end{array}
\]

\[
\varepsilon = 748.9 \text{ (k}^2 \text{ft)}
\]

\[
Q^2 \delta_D = \sum \frac{F_P F_Q L}{AE} = \frac{(748.9)(12)}{(5)(29000)} = 0.062''
\]

b) \[
Q^2 \delta_D = (F_Q F_P) \alpha \Delta T L_{st} \quad (\text{Assume } \alpha = 1 \text{ if desired})
\]

\[
1 \frac{k}{(0.062'')} = (-2) \alpha \Delta T (10') \quad (12 \text{ in/ft})
\]

\[
\Delta T = 2.58 \cdot 10^{-4}
\]
Problem 4.) Influence Lines

a.) For the structure below, sketch the influence line diagram for the moment at A. (15 points)

b.) Trick Question: For the structure above, sketch the influence line diagram for the moment at B. (5 points)

("Sketch" indicates that numbers are not required, only qualitative shapes.)

a.)

b.) (No internal moment allowed at hinge. Influence line diagram is zero at all locations.)