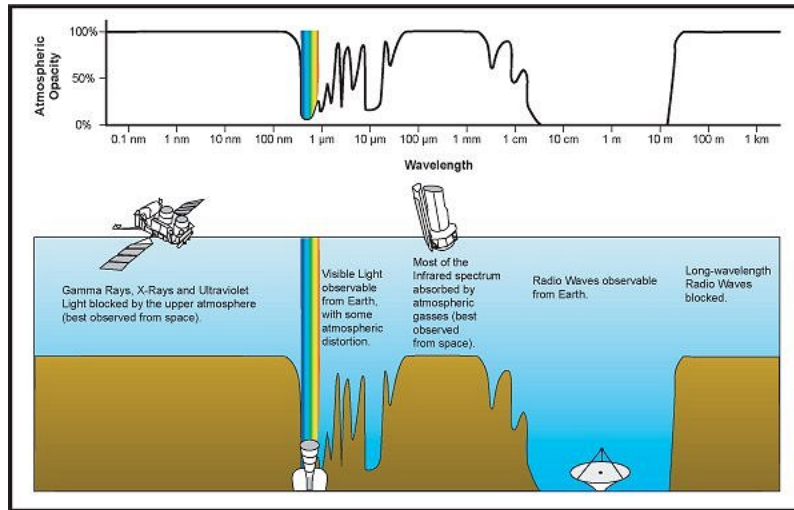


Atmospheric Transmission



Beer's Law

- The rate of power attenuation per unit distance is given by the **absorption coefficient** β_a (with dimensions of inverse length), related to n_i and wavelength λ by:

$$\beta_a = \frac{4\pi n_i}{\lambda}$$

- For an initial intensity I_0 at position $x = 0$ in a homogeneous medium, propagating in the x -direction:

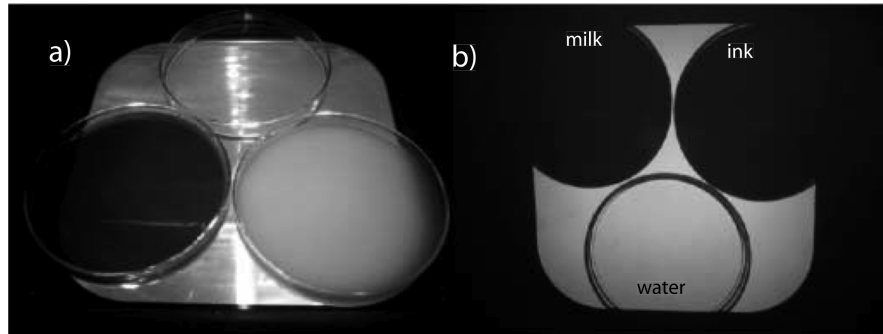
$$I_\lambda(x) = I_{\lambda,0} e^{-\beta_a x}$$

$$\Rightarrow \frac{I_\lambda(x)}{I_{\lambda,0}} = e^{-\beta_a x} = t_\lambda(x)$$

- Where t is the **transmittance**: the fraction of radiation that survives the trip over the distance x (i.e., that which is not absorbed)
- To generalize Beer's Law for the atmosphere, we need to account for more than just absorption...

Extinction of radiation

Milk, ink and water in dishes on an overhead projector

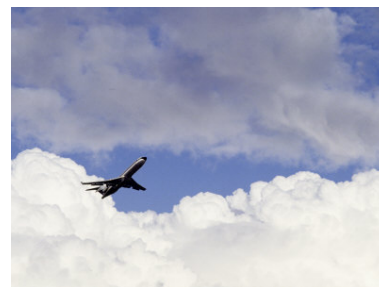


a) Top view (reflected light)

b) Projected view (transmitted light)

- Dark shadows for milk (scattering medium) and ink (absorbing medium) show that *absorption* and *scattering* are equally effective at depleting transmitted radiation

Black clouds are not absorbing...



- The appearance of clouds depends on whether they are viewed by reflected (scattered) or transmitted light
- In the latter case, clouds appear dark against a bright background due to attenuation (scattering) of light by the cloud (though only for thick clouds)

Extinction of radiation

- A beam of radiation can be attenuated (*extinguished*) not only by **absorption** but also by **scattering**
- The reduction of intensity of a beam of radiation due to absorption and scattering is called **extinction**
- Hence we define a more general **extinction coefficient** (β_e) to replace the absorption coefficient (β_a) encountered earlier

$$\beta_e = \beta_a + \beta_s$$

...we also introduce a scattering coefficient (β_s)

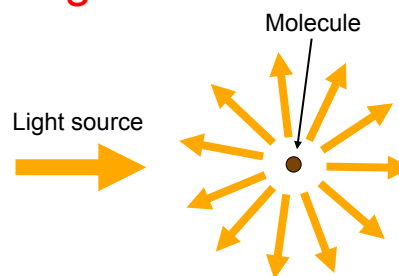
What is scattering?

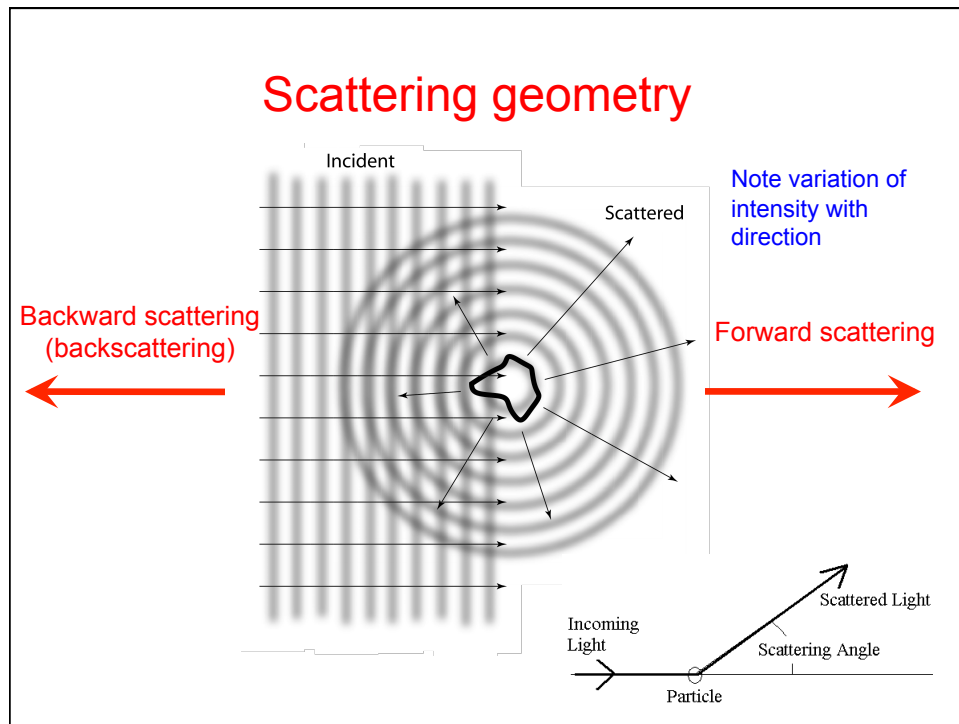
When light encounters matter, matter not only re-emits light in the forward direction, but it also re-emits light in all other directions.

This is called **scattering** – the redirection of radiation out of the original direction of propagation.

Contrast with absorption, which involves conversion of EM energy to heat or chemical energy.

Light scattering is everywhere. All molecules scatter light. Surfaces scatter light. Scattering causes milk and clouds to be white and the daytime sky to be blue. It is the basis of nearly all optical phenomena.





Thin clouds – effect of scattering



- Optically thin clouds can look brighter when viewed towards the Sun, and vice versa
- This is due to the extreme asymmetry in forward-backward scattering by cloud droplets, and the dominance of single (not multiple) scattering

Extinction of radiation

- $\beta_e = \beta_a + \beta_s$

- The **extinction coefficient** (β_e) is the sum of an absorption coefficient (β_a) and a scattering coefficient (β_s) – all have units of inverse length (m^{-1})

- For milk, $\beta_a \approx 0$ so $\beta_e \approx \beta_s$; for ink $\beta_s \approx 0$ so $\beta_e \approx \beta_a$

- To characterize the relative importance of scattering and absorption in a medium, the **single scatter albedo** (ω) is used:

$$\omega = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_s + \beta_a}$$

- $\omega = 0$ for a purely absorbing medium (ink), 1 in a purely scattering medium (milk)

Based on their appearance what would you conclude about the spectral dependence of β_e and ω for.....

- A cloud?

Highly reflective of sunlight, suggesting that the SSA is very close to one. Clouds are white, rather than some other shade, suggesting that the SSA and extinction coefficient don't depend much on wavelength within the visible band.



Based on their appearance what would you conclude about the spectral dependence of β_e and ω for.....

- Diesel exhaust fumes?

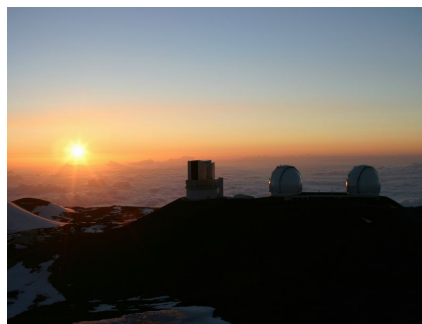
Scatters some light, but appear rather dark gray, suggesting that the SSA is greater than zero but much less than one. Color is rather neutral, so spectral dependence is small.



Based on their appearance what would you conclude about the spectral dependence of β_e and ω for.....

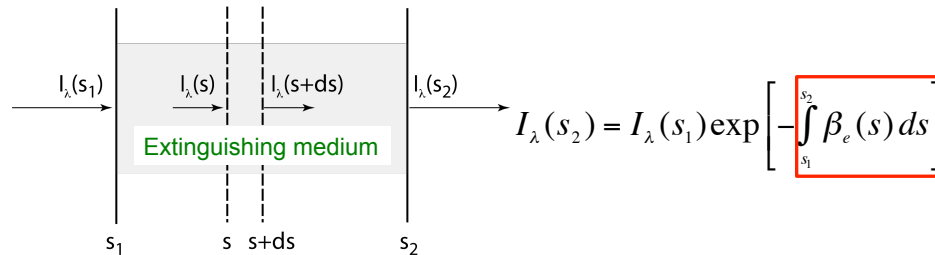
- A cloud-free atmosphere?

The setting sun (which is seen via a rather long path through the atmosphere) is reddish, suggesting that extinction is greater for shorter wavelengths. Much of the radiation that is extinguished is seen in other directions as scattered radiation -- i.e., the blue sky; hence, we conclude that the SSA is reasonably close to unity.



Extinction over a finite path

- Need to generalize Beer's Law for a path through the atmosphere, where β_e varies with location, and the propagation direction is arbitrary.



$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \beta_e(s) ds$$

The integral quantity is the *optical depth* or *optical thickness* (when measured vertically in the atmosphere) or the *optical path*. It can have any positive value. **What are its dimensions?**

We also can also write *transmittance* (t) as: $t(s_1, s_2) = e^{-\tau(s_1, s_2)}$

Properties of Beer's Law

- Transmittance is a dimensionless quantity ranging from near zero to one.
- If β_e is constant along the path (between s_1 and s_2) then:

$$\tau = \beta_e (s_2 - s_1)$$

- Each dimensionless unit of optical depth corresponds to a reduction of I_λ to e^{-1} or $\sim 37\%$ of its initial value.
- For propagation of radiation along an extended path from s_1 to s_N , consisting of sub-paths s_1 to s_2 , s_2 to s_3 , s_{N-1} to s_N etc.,

$$\tau(s_1, s_N) = \tau(s_1, s_2) + \tau(s_2, s_3) + \cdots + \tau(s_{N-1}, s_N)$$

$$t(s_1, s_N) = t(s_1, s_2) \times t(s_2, s_3) \times \cdots \times t(s_{N-1}, s_N)$$

- i.e., **the total optical depth is the sum of the individual optical depths, and the total transmittance is the product of the individual transmittances**

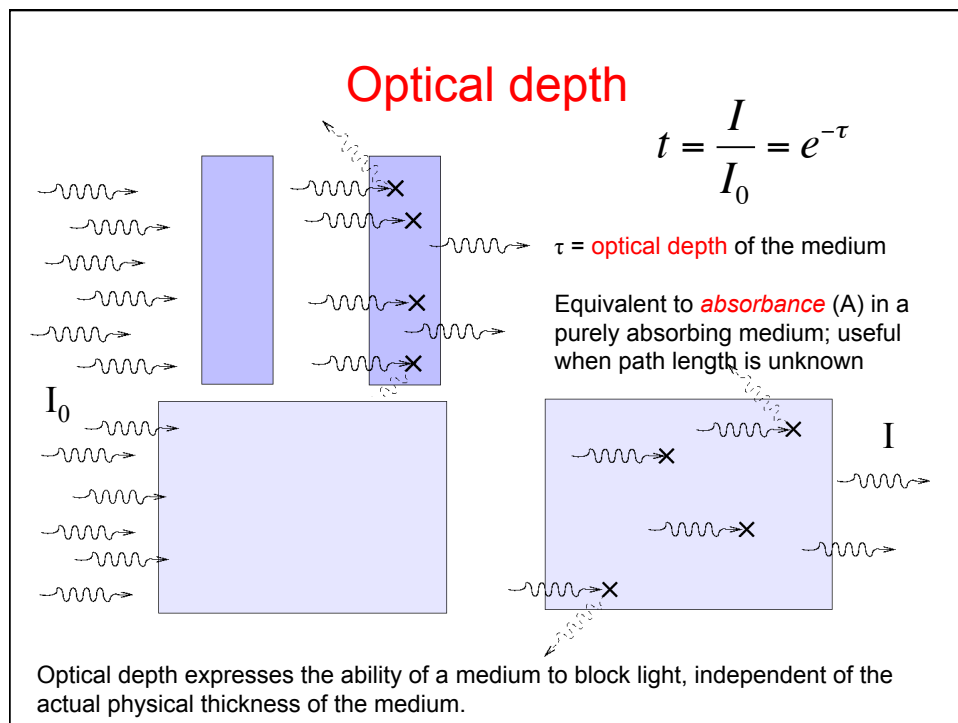
Properties of Beer's Law

- For propagation of radiation over an optical path $\tau \ll 1$ (i.e., a transparent medium), the transmittance can be approximated by:

$$t = \exp(-\tau) \approx 1 - \tau(s_1, s_2) = 1 - \beta_e(s_2 - s_1)$$

- If a medium doesn't scatter ($\omega = 0$), then whatever is not transmitted along a given path must be absorbed. In this case, the path *absorbance* or *absorbance (a)* is:

$$a = 1 - t$$



Optical depth

The optical depth expresses the *quantity of light removed from a beam by absorption or scattering* during its path through a medium.

If $\tau = 0.5$, transmissivity = ~60%, if $\tau = 1$, $t = \sim 37\%$ and if $\tau = 2.5$, $t = \sim 8\%$.

If $\tau \ll 1$, the medium is **optically thin**

If a gas is optically thin, then the chances are small that a photon will interact with a single particle, and VERY small that it will interact with more than one, i.e. we can ignore multiple scatterings or absorptions. In general terms, we can see right through the cloud.

In the optically thin regime, the amount of **extinction** (absorption plus scattering) is linearly related to the amount of material. Hence if we can measure the amount of light absorbed by the gas, we can calculate exactly how much gas there is.



Optical depth

If $\tau \gg 1$, the medium is **optically thick**

If a gas is optically thick, a photon will interact many, many times with particles before it finally escapes from the cloud. Any photon entering the cloud will have its direction changed many times by scattering – so its ‘output’ direction has nothing to do with its ‘input’ direction. We can’t see anything at all through the cloud: *it is opaque*.

You can’t see through an optically thick medium; you can only see light emitted by the very outermost layers. But there is one convenient feature of optically thick materials: *the spectrum of the light they emit is a blackbody spectrum*, or very close to it.



Optical depth problem

The optical depth of the stratospheric aerosol layer is typically about 1×10^{-4} for wavelengths of $1 \mu\text{m}$. Following the eruption of Mt. Pinatubo volcano in June 1991, the optical depth at $1 \mu\text{m}$ increased to 1×10^{-2} . How much more or less infrared radiation was transmitted through this layer? What do you think the consequences of this change might be?



Pinatubo eruption, June 1991

Space Shuttle view of aerosol veil, August 1991

Optical depth problem

From the restatement of Beer's Law, we know that $I/I_0 = \exp(-\tau)$, where the ratio I/I_0 represents the fraction of light transmitted through the medium.

We can make use of an approximation: for small values of x , $e^{-x} \approx (1 - x)$. Both values for the optical depth are quite small, so we can approximate:

$$\text{Typical: } I/I_0 = e^{-(0.0001)} \approx (1 - 0.0001) \approx 1$$

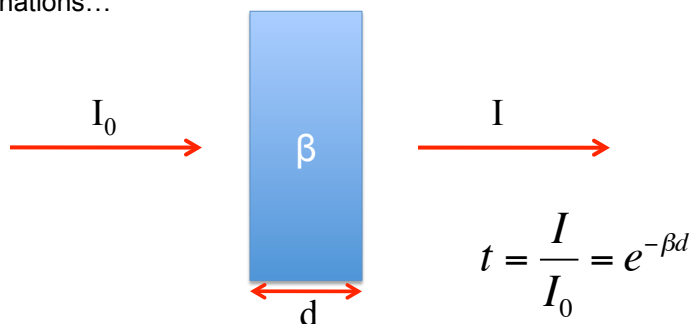
$$\text{Post-volcano: } I/I_0 = e^{-(0.01)} \approx (1 - 0.01) \approx 0.99$$

The amount of light transmitted through the aerosol layer decreased following the volcanic eruption because the optical depth increased. Approximately 1% less light was transmitted. That missing 1% had to go somewhere - either be scattered by the particles or absorbed by them. In this case, it was likely absorbed.

Evidence has shown that the enhanced aerosol layer following the Pinatubo eruption was responsible for slightly increasing the temperature of the lower stratosphere..

More Beer's Law

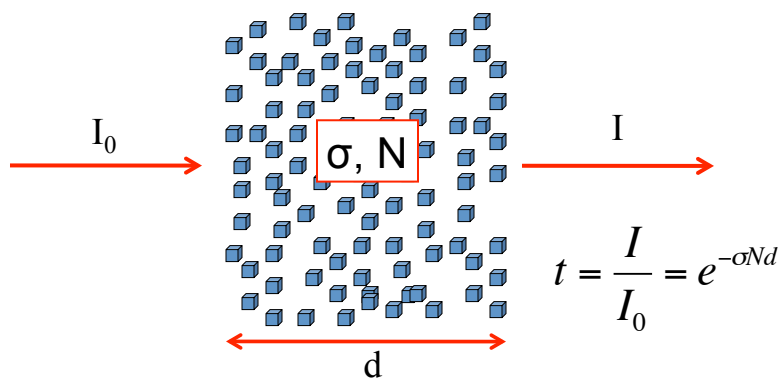
Also known as the Beer-Lambert Law or the Beer-Bouguer-Lambert Law, or other combinations...



The law states that there is a logarithmic dependence between the transmission (or transmissivity), t , of light through a substance and the product of the absorption coefficient (β) and the distance the light travels through the material, or the path length (d).

More Beer's Law

For a gaseous absorber, the absorption coefficient (β) is written as the product of an **absorption cross-section** (σ , cm^2) and the **number density of absorbers** (molecules cm^{-3}):



(For liquids the absorption coefficient is the product of the molar absorptivity of the absorber and the concentration of the absorber)

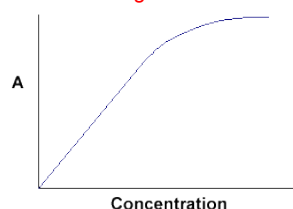
More Beer's Law

Now taking the natural logarithm (the convention for atmospheric science):

$$\Rightarrow \ln\left(\frac{I}{I_0}\right) = -\sigma Nd$$

$$\Rightarrow A = -\ln\left(\frac{I}{I_0}\right) = \ln\left(\frac{I_0}{I}\right) = \sigma Nd$$

Deviation at high concentrations



Where A is the **absorbance** (negative logarithm of transmittance).

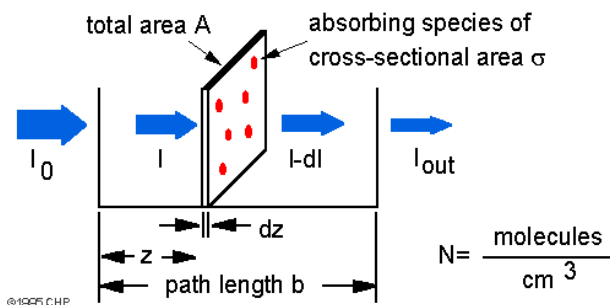
100% transmittance = absorbance of zero; 10% transmittance = absorbance of 2.3

Hence Beer's Law states that there is a **linear relationship between absorbance and concentration** (as opposed to a logarithmic dependence on transmittance).

If the path length (d) and absorption cross-section are known and the absorbance is measured, **the number density of absorbers can be deduced**.

Derivation of Beer's Law

The Beer-Lambert law can be derived by approximating the absorbing molecule by an opaque disk whose cross-sectional area, σ , represents the effective area seen by a photon of wavelength λ . Taking an infinitesimal slab, dz , of sample:



I_0 is the intensity entering the sample at $z = 0$, I_z is the intensity entering the infinitesimal slab at z , dI is the intensity absorbed in the slab, and I is the intensity of light leaving the sample.

Then, the total opaque area on the slab due to the absorbers is: $\sigma * N * A * dz$.

The fraction of photons absorbed is therefore: $\sigma * N * A * dz / A = \sigma N dz$

Derivation of Beer's Law

The fraction of photons absorbed by the slab is: $\frac{dI}{I_z} = -\sigma N dz$
(negative since photons are removed)

Integrating both sides gives: $\ln I_z = -\sigma N z + C$

The difference in intensity between $z = 0$ and $z = b$ is therefore:

$$\ln I_0 - \ln I = (-\sigma N 0 + C) - (-\sigma N b + C) = \sigma N b$$

$$\Rightarrow \ln\left(\frac{I_0}{I}\right) = A = \sigma N b$$

Or for liquids: $A = \log_{10}\left(\frac{I_0}{I}\right) = \frac{\sigma N b}{2.303} = \epsilon c b$

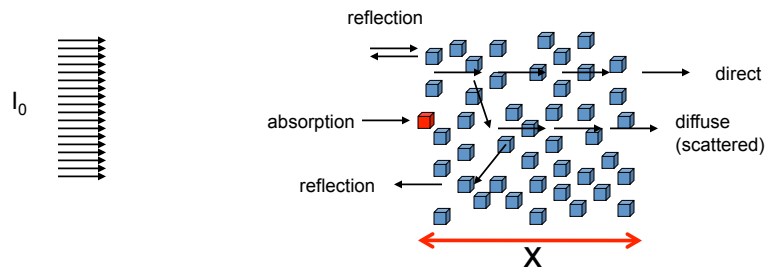
Where ϵ = molar absorptivity ($\text{L mol}^{-1} \text{cm}^{-1}$ or $\text{mol}^{-1} \text{cm}^2$) and c = concentration (mol cm^{-3})

Deviations from Beer's Law

At least five conditions need to be satisfied in order for Beer's law to be valid. These are:

- The absorbers must act independently of each other.
 - The absorbing medium must be homogeneously distributed in the interaction volume and must not scatter the radiation.
 - The incident radiation must consist of parallel rays, each traversing the same length in the absorbing medium.
 - The incident radiation should preferably be monochromatic, or have at least a width that is narrower than the absorbing transition.
 - The incident flux must not influence the atoms or molecules; it should only act as a non-invasive probe of the species under study.
- If any of these conditions is not fulfilled, there will be deviations from Beer's law.

Deviations from Beer's Law



Direct Beam:

$$I(x) = I_0 \exp(-\beta_{\text{ext}} x)$$

Transmission Coefficient for the Direct Beam:

$$t_{\text{direct}} = \exp(-\beta_{\text{ext}} x)$$

Beer's Law applies to the direct beam only

Mass extinction coefficient

- The extinction coefficient β_e expresses extinction by reference to a geometric distance (or path length) through a medium
- **Problem:** The measured transmission through a ink-water solution is 70% and the depth of the solution is 10 cm. Calculate β_e .
- **What happens if the depth of the solution is doubled without addition of ink?**
- In remote sensing, we are usually interested in the mass of absorbing material present, or the number of absorbing particles.
- We define the **mass extinction coefficient** k_e , relating the density of the absorbing material (ρ) to β_e :

$$\beta_e = \rho k_e$$

Mass extinction coefficient

- The density of the absorber is given by: $\rho = \frac{M}{HA}$

Where M is the mass of absorber in the solution, H is the depth of the solution, and A is the horizontal area of the 'container'.

- The transmittance is then: $t = \exp(-\tau) = \exp(-\beta_e H)$
 $= \exp\left[-k_e \left(\frac{M}{HA}\right) H\right] = \exp\left[-k_e \left(\frac{M}{A}\right)\right]$
 $\Rightarrow \tau = k_e \frac{M}{A}$

- So the transmittance does not depend on H, only M and A.

Extinction cross-section

- The dimensions of k_e are *area per unit mass*, or *extinction cross-section per unit mass*
- The extinction cross-section is used when considering molecules of an absorbing gas, droplets of water in a cloud, or soot particles in a smoke plume.
- We use the *number density* ($N; m^{-3}$) to characterize the particle concentration.
- To link the extinction coefficient and N we can write: $\beta_e = \sigma_e N$
- Where the *constant of proportionality* σ_e has dimensions of area and is the *extinction cross-section*; or the effective area of the particle 'blocking' transmission of radiation

Extinction efficiency

- We also define the extinction efficiency Q_e :

$$Q_e = \frac{\sigma_e}{A}$$

Where A is the geometric cross-sectional area of the particle, which for a spherical particle of radius r is equal to πr^2

- Should Q_e only range from zero to one?

Generalization to scattering and absorption

$$\beta_a = \rho k_a = \sigma_a N \quad \beta_s = \rho k_s = \sigma_s N$$

- Similar quantities can also be defined for absorption and scattering
- k_a and k_s are the *mass absorption* and *mass scattering coefficients*
- σ_a and σ_s are the *absorption* and *scattering cross-sections*

$$\sigma_a = Q_a A \quad \sigma_s = Q_s A$$

- Q_a and Q_s are the *absorption* and *scattering efficiencies*, where A is the cross-sectional area of a particle
- The single scatter albedo (ω) can also be defined using these quantities

Generalization to mixtures of components

- Real particles in the atmosphere are complex, and many different gases and aerosol types are present...



$$\beta_e = \sum_i \beta_{e,i} = \sum_i \rho_i k_{e,i} = \sum_i \sigma_{e,i} N_i$$

The total extinction, scattering and absorption coefficients for a mixture of components = sum of the corresponding coefficients for each individual component

Plane parallel approximation

- The atmosphere is usually approximated as *plane-parallel* in atmospheric remote sensing (also called *slab geometry*)
- Ignore horizontal variations and the curvature of the earth

$$\frac{H}{\cos \theta} \ll R$$

Smoke layer from forest fire

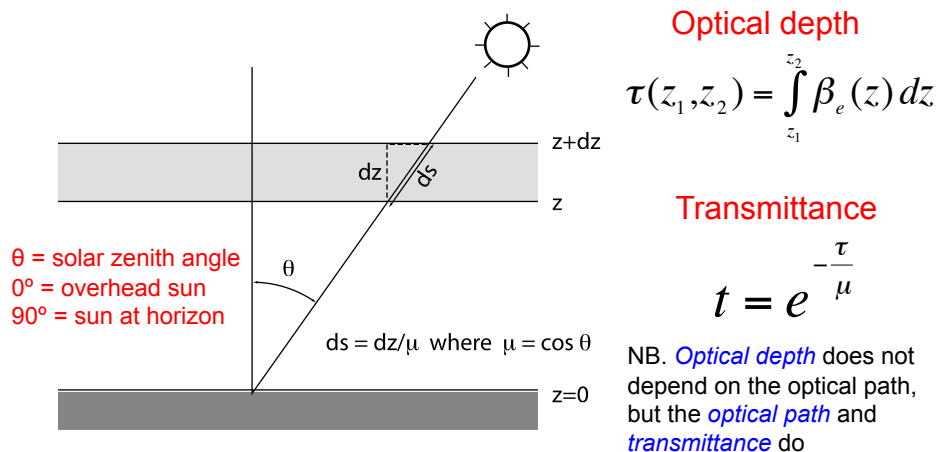


Exception!
Limb measurements

Volcanic aerosol layer

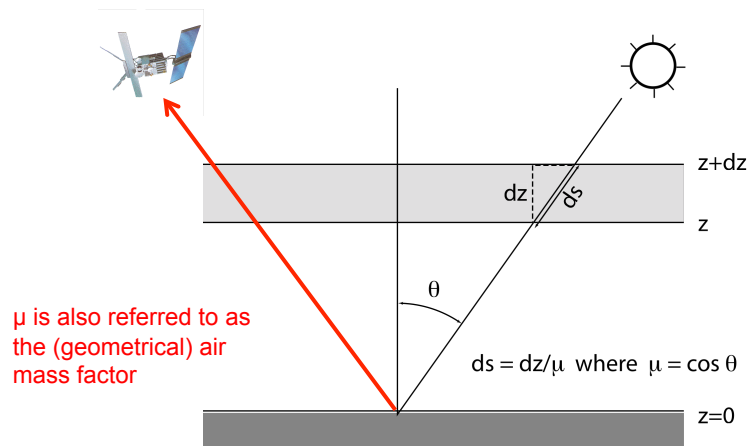


Slant and vertical atmospheric paths



- Remote sensing is often done when the sun is not directly overhead, so the radiation follows a **slant path** rather than a **vertical path**

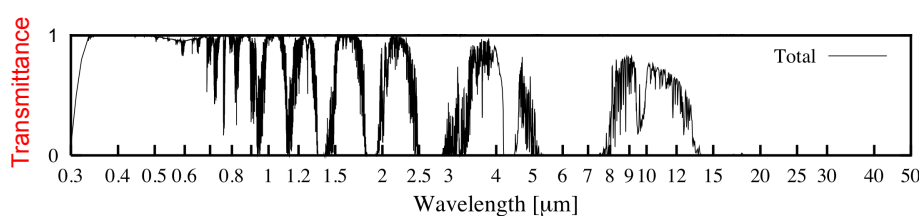
Satellite viewing geometry



- Satellites measuring reflected and scattered radiation (UV/visible/SWIR) detect radiation that has been transmitted twice through the atmosphere

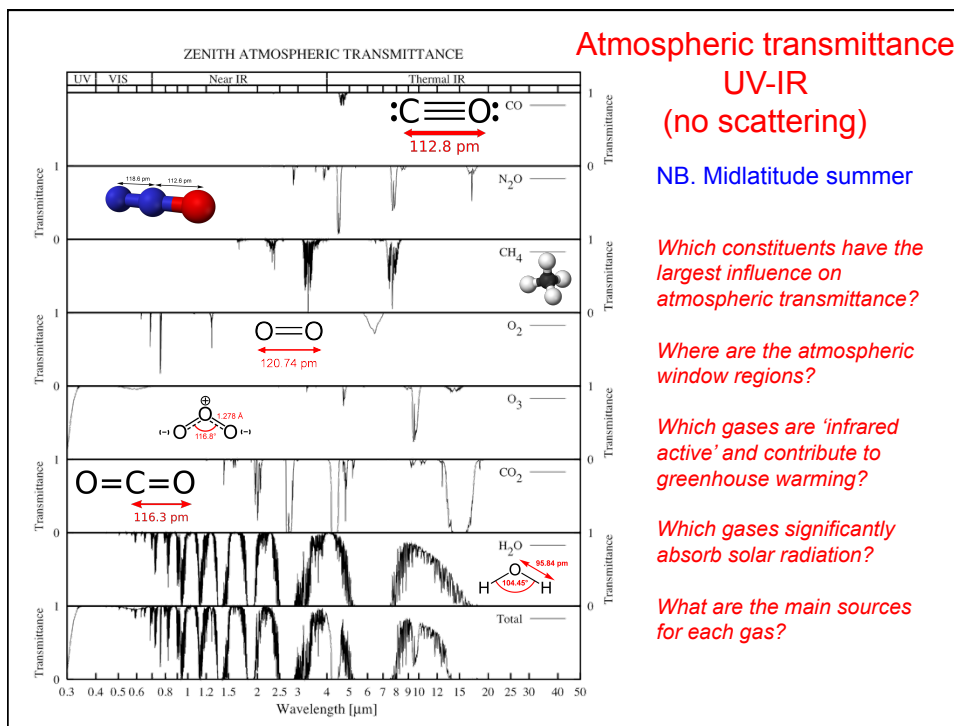
Transmission spectrum of the atmosphere

- At which wavelengths is the cloud-free atmosphere relatively transparent? (*Atmospheric windows*)
- At which wavelengths is the cloud-free atmosphere strongly absorbing, and which constituents are responsible for the absorption?
- How do the extinction and scattering properties of clouds vary with wavelength?



Atmospheric composition

Constituent	Fraction (by volume) in dry air	Significant absorption bands	Remarks
Nitrogen, N ₂	78.1%	-	
Oxygen, O ₂	20.9%	UV-C, MW near 60 and 118 GHz, weak in VIS and IR	
H ₂ O	0-2 %	Numerous strong bands in IR; also MW near 183 GHz	Highly variable in time and space
Ar & inert gases	0.936%	-	monoatomic
CO ₂	370 ppm	Near 2.8, 4.3, and 15 μm	Increasing 1.6 ppm/year
Methane, CH ₄	1.7 ppm	Near 3.3 and 7.8 μm	Increasing
Nitrous oxide, N ₂ O	0.35 ppm	4.5, 7.8 and 17 μm	
Carbon monoxide, CO	0.07 ppm	4.7 μm (weak)	
Ozone, O ₃	~10 ⁻⁸	UV-B, 9.6 μm	



Atmospheric transmittance UV-IR (no scattering)

NB. Midlatitude summer

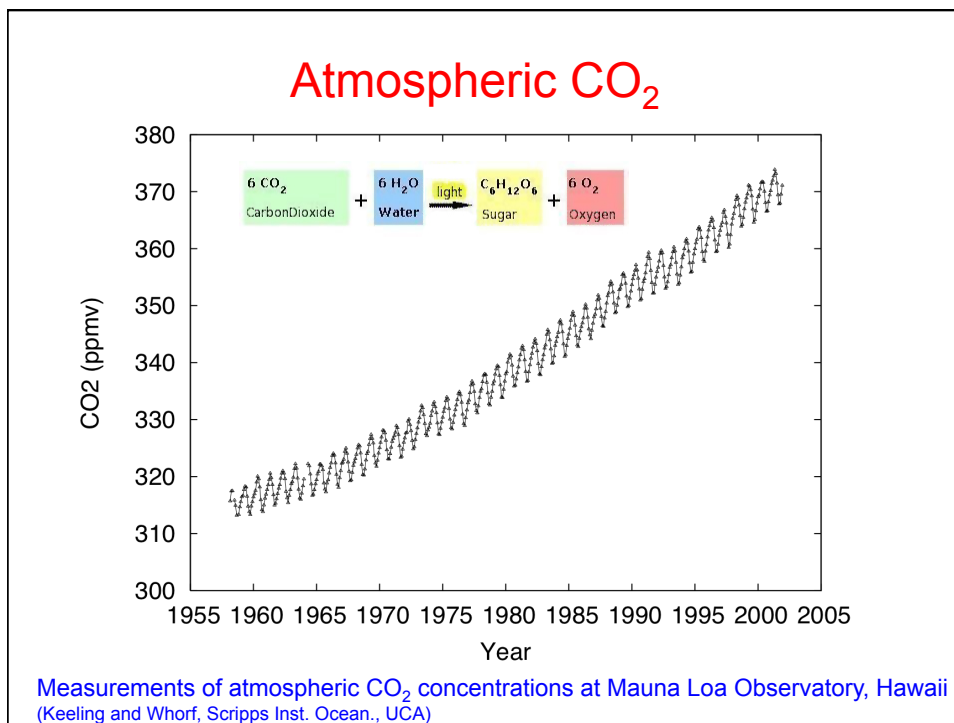
Which constituents have the largest influence on atmospheric transmittance?

Where are the atmospheric window regions?

Which gases are 'infrared active' and contribute to greenhouse warming?

Which gases significantly absorb solar radiation?

What are the main sources for each gas?

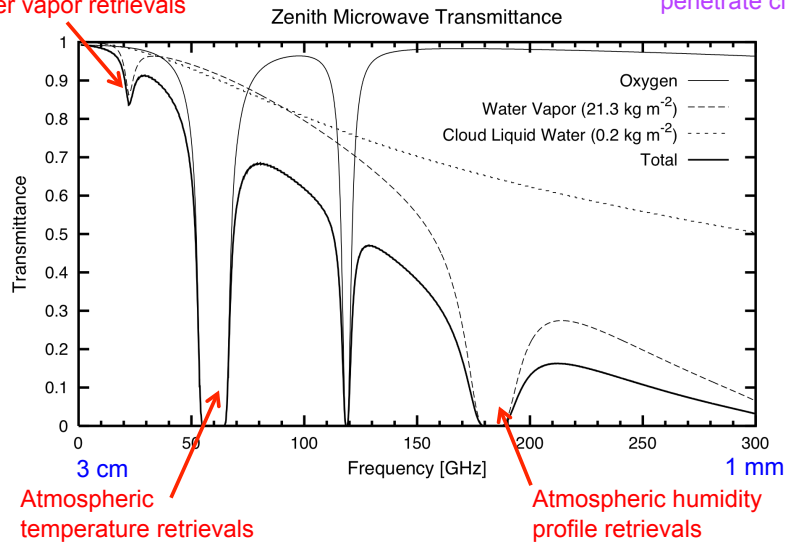


Microwave transmittance

Total atmospheric water vapor retrievals

NB. Midlatitude conditions

Microwaves penetrate clouds!

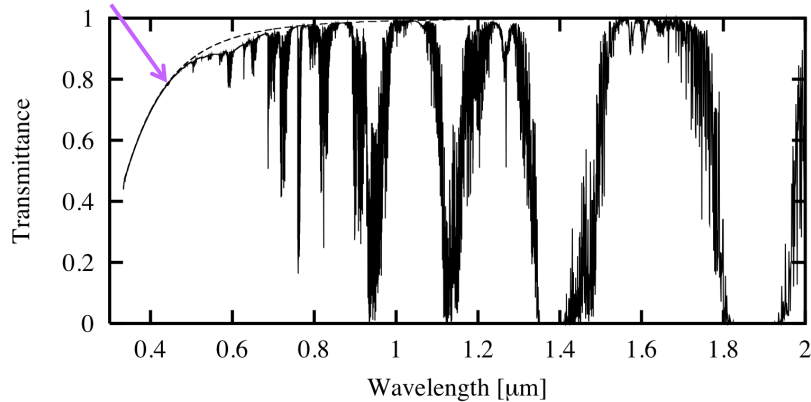


Effects of scattering

NB. Midlatitude summer conditions

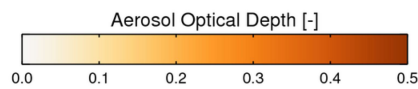
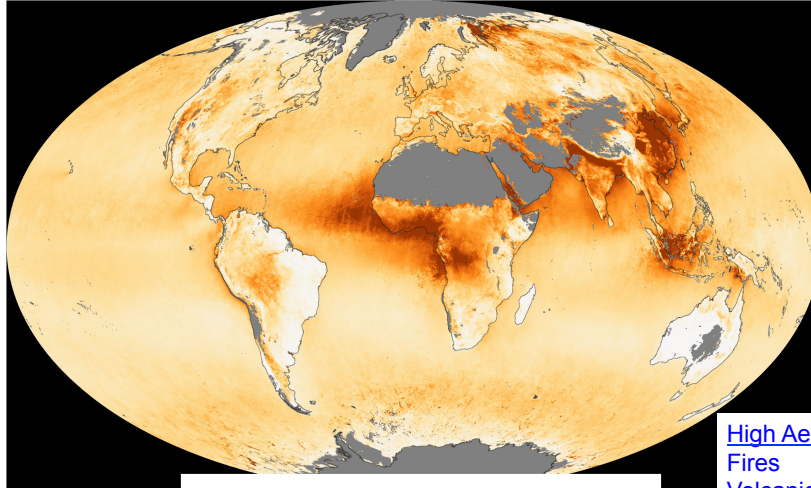
Scattering

Zenith Atmospheric Transmittance (SW)



The scattering cross-section (σ_s) of an air molecule is proportional to λ^{-4}
How much stronger is scattering at 0.4 μm compared to 0.7 μm ?

Extinction and scattering by aerosols and clouds



High Aerosol AOD:
 Fires
 Volcanic eruptions
 Dust storms
 Severe pollution

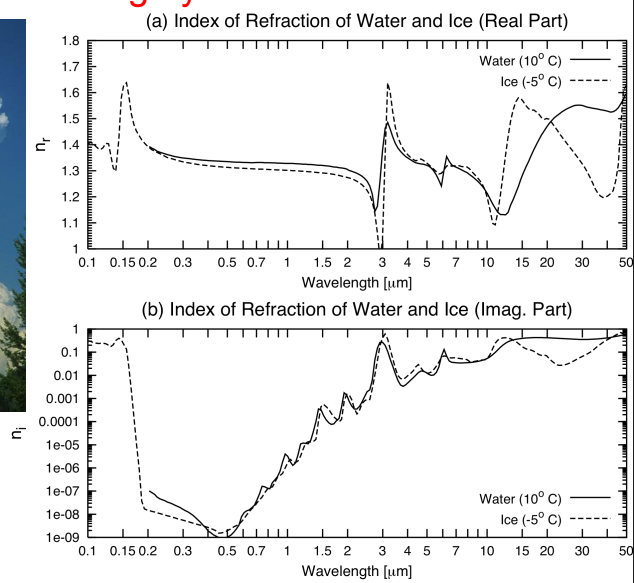
2006 Annual Mean Aerosol Optical Depth (AOD) at 550 nm from MODIS

Extinction and scattering by aerosols and clouds



Where do clouds only scatter radiation ($\omega = 1$)?

Where could ice and water clouds be distinguished?

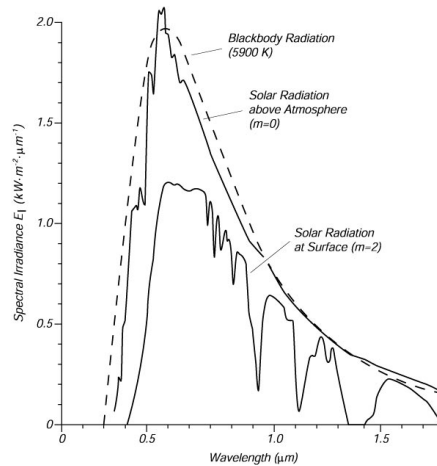


Measuring solar intensity from the ground

- We know that the solar irradiance at the top of the atmosphere (S_0) is of crucial importance to Earth's radiation budget

- Satellite sensors now measure the solar flux and solar spectrum, but how was this done before the first satellites, using ground-based measurements?

- **Problem:** solar intensity measured at ground level is reduced by atmospheric absorption and scattering, but the atmospheric transmittance is unknown. But the latter cannot be inferred without knowing the solar flux – hence a 'Catch 22' situation.



Measuring solar intensity from the ground

- **Solution:**

- Assume a plane-parallel atmosphere with constant properties over the course of a day (i.e., a clear day with stable temperature, pressure)

- For any given wavelength (λ), there are 2 unknowns: the solar intensity (S_λ) and the atmospheric optical depth (τ_λ)

- From Beer's Law, the intensity of solar radiation measured at sea level (I_λ) is:

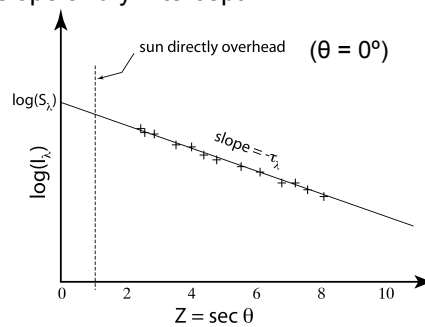
$$I_\lambda = S_\lambda \exp\left(-\frac{\tau_\lambda}{\mu}\right) \quad \mu = \cos \theta \text{ and } \theta = \text{solar zenith angle}$$

Measuring solar intensity from the ground

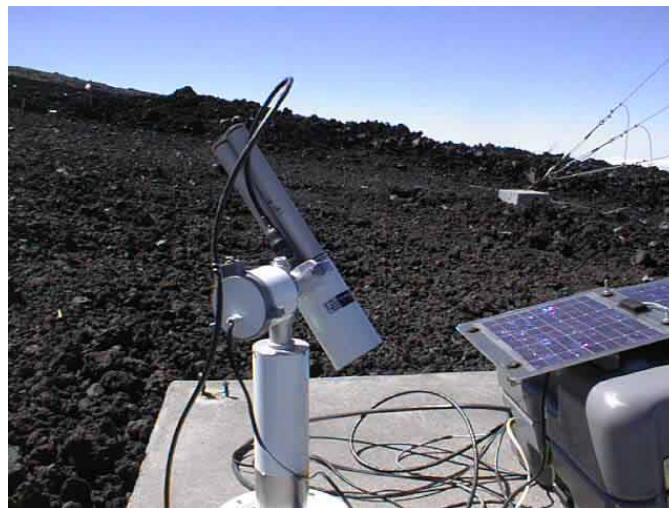
- Now take the logarithm of both sides:

$$\ln(I_\lambda) = -\frac{\tau_\lambda}{\mu} + \ln(S_\lambda)$$

- Define $x = 1/\mu = \sec \theta$ then this is a linear equation of the form $y = mx+c$, where $y = \ln(I_\lambda)$, the slope $m = -\tau_\lambda$ and the y-intercept $c = \ln(S_\lambda)$.
- Hence to determine τ_λ and S_λ , measure y for a range of values of x , i.e. at different times of day as the sun rises, reaches its zenith, and sets. Plot the data and find the slope and y-intercept.



Sun photometer



<http://aeronet.gsfc.nasa.gov/>

Global network of sun photometers measuring aerosol optical depth
Calibrated using *Langley technique* at Mauna Loa Observatory, Hawaii

The exponential atmosphere

- The density of the atmosphere decays exponentially with height z :

$$\rho(z) = \rho_0 e^{-\frac{z}{H}}$$

- Where ρ_0 is the density at sea level and H (≈ 8 km) is the *scale height* (the altitude change that leads to a factor e change in density)
- So for a 'well-mixed' constituent (like CO_2), its density is:

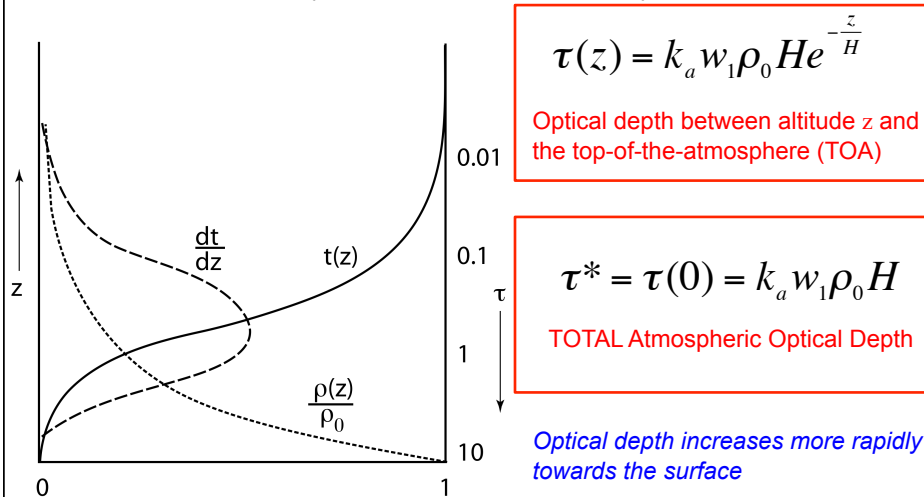
$$\rho_1(z) = w_1 \rho_0 e^{-\frac{z}{H}}$$

- where w_1 is the *mixing ratio* (mass of constituent per unit mass of air)
- Assume a mass absorption coefficient k_a for the constituent that depends on λ but not T or P , and a nonscattering atmosphere at the λ of interest:

$$\beta_e(z) = k_a w_1 \rho_0 e^{-\frac{z}{H}}$$

Optical depth in an exponential atmosphere

$$\tau(z) = \int_z^{\infty} \beta_e(z') dz' = k_a w_1 \rho_0 \int_z^{\infty} e^{-\frac{z'}{H}} dz'$$

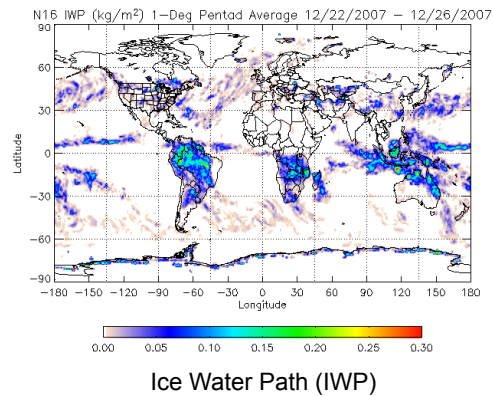


Mass path

$$\tau(z) = k_a w_1 \rho_0 \int_z^\infty e^{-\frac{z'}{H}} dz' = k_a u(z)$$

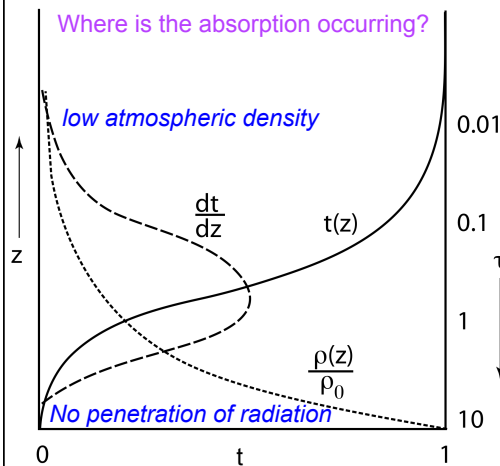
$$u(z) = \int_z^\infty \rho_1(z') dz'$$

- **Mass path** of the constituent between altitude z and the top-of-the-atmosphere (TOA)
- Dimensions: mass per unit area (e.g., $g\ m^{-2}$)
- Integral over all z for water vapor gives an important atmospheric quantity: *total precipitable water, water vapor burden, water vapor path, liquid water path, cloud water path....(also for ice)*



Transmittance in an exponential atmosphere

$$t(z) = \exp\left[-\frac{\tau(z)}{\mu}\right] = \exp\left[-\frac{k_a w_1 \rho_0 H}{\mu} e^{-\frac{z}{H}}\right]$$



$$W(z) = \frac{dt(z)}{dz} = \frac{\beta_e(z)}{\mu} t(z)$$

The local rate of absorption within the atmosphere ($W(z)$) equals the local rate of change of transmittance from level z to the TOA, or the local extinction coefficient at level z times the transmittance from z to the TOA

$W(z)$ is called the *absorption weighting function*. Its peak depends on k_e at the wavelength in question, but occurs where $\tau(z) = 1$

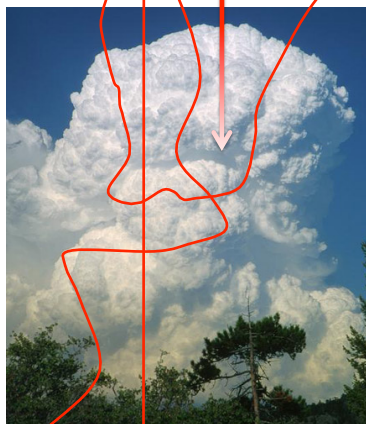
Optical thickness and transmittance of clouds

Can you see the sun through a cloud?

Liquid water clouds

Droplet radii: 5-15 μm

$N = 10^2\text{-}10^3 \text{ cm}^{-3}$



$$t_{\text{dir}} + t_{\text{diff}} + r + a = 1$$

$$t + r + a = 1$$

$$t = t_{\text{dir}} + t_{\text{diff}}$$

t_{diff} (diffuse transmittance)

t_{dir} (direct transmittance)

r = reflected to space

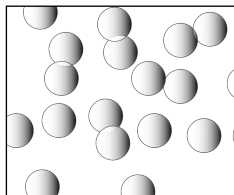
t = available for surface heating

a = heats atmosphere locally

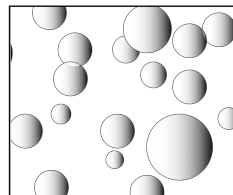
Optical thickness and transmittance of clouds

- t_{dir} , t_{diff} , r and a depend on the cloud optical thickness (τ^*), single-scatter albedo (ω) and how radiation is scattered by the cloud droplets
- Values of t_{diff} , r and a depend on *multiple scattering* of radiation by the cloud droplets

Monodisperse



Polydisperse



- Consider a **monodisperse** cloud composed of droplets of identical radius (r) with a concentration $N \text{ m}^{-3}$
- Volume extinction coefficient:

$$\beta_e = N Q_e \pi r^2$$

Optical thickness and transmittance of clouds

- Note that N and r are difficult to measure, but we can measure or estimate the cloud water density ρ_w (typically $0.1\text{-}1\text{ g m}^{-3}$)
- For a *monodisperse* cloud, the cloud water density is N times the mass of water in each droplet:

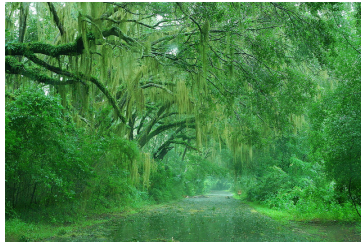
$$\rho_w = N \frac{4}{3} \pi r^3 \rho_l$$

- where ρ_l is the density of pure water ($\sim 1000\text{ kg m}^{-3}$)
- This gives us a new expression for the volume extinction coefficient:

$$\beta_e = N Q_e \pi r^2 = k_e \rho_w = k_e N \frac{4}{3} \pi r^3 \rho_l$$

- And solving for k_e gives: $k_e = \frac{3Q_e}{4\rho_l r}$ Note inverse dependence on r !

Rain and fog



$Q_e \approx 2$ at visible wavelengths

$$r \approx 1\text{ mm}$$

$$k_e = 1.5\text{ m}^2\text{ kg}^{-1}$$

$$\rho_w = 0.1\text{ g m}^{-3}$$

$$\beta_e = 0.15\text{ km}^{-1}$$

$$k_e = \frac{3Q_e}{4\rho_l r}$$

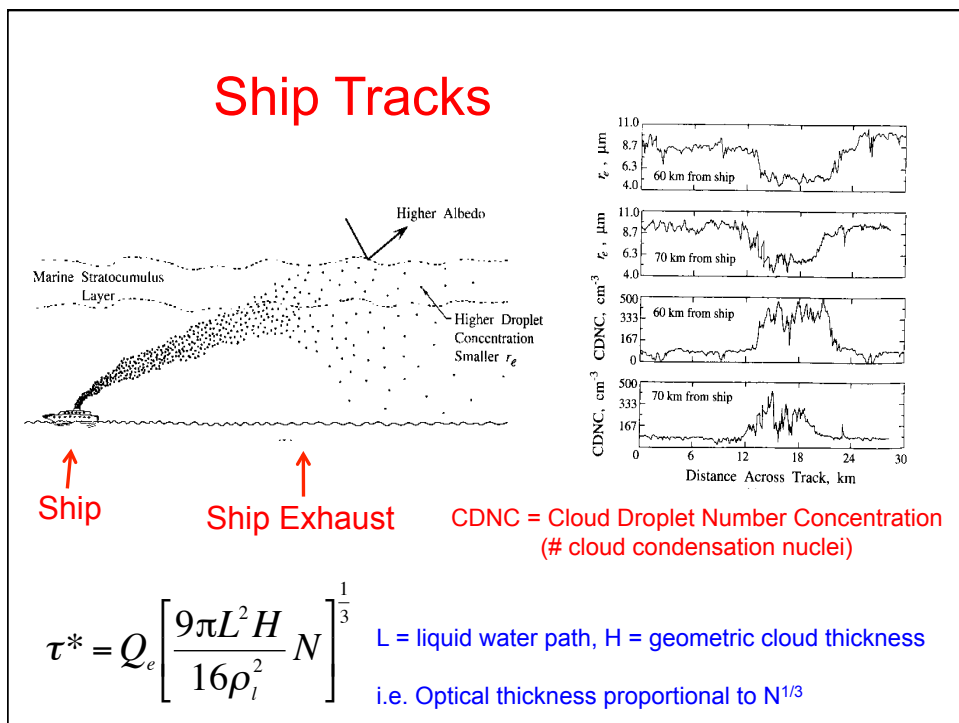
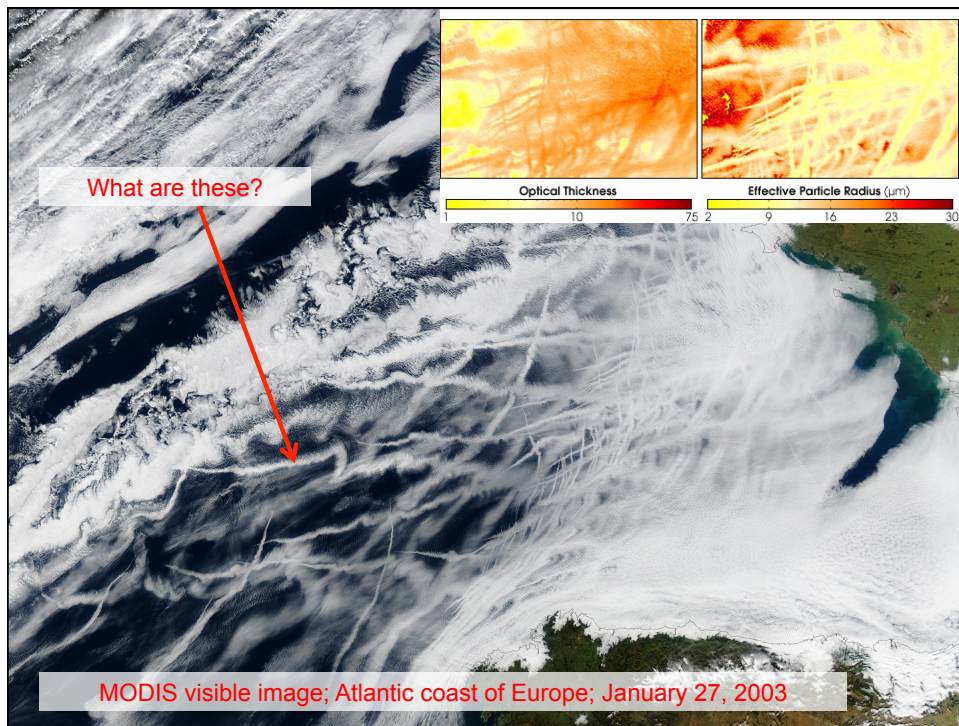
$$\beta_e = k_e \rho_w$$

$$r \approx 10\text{ }\mu\text{m}$$

$$k_e = 150\text{ m}^2\text{ kg}^{-1}$$

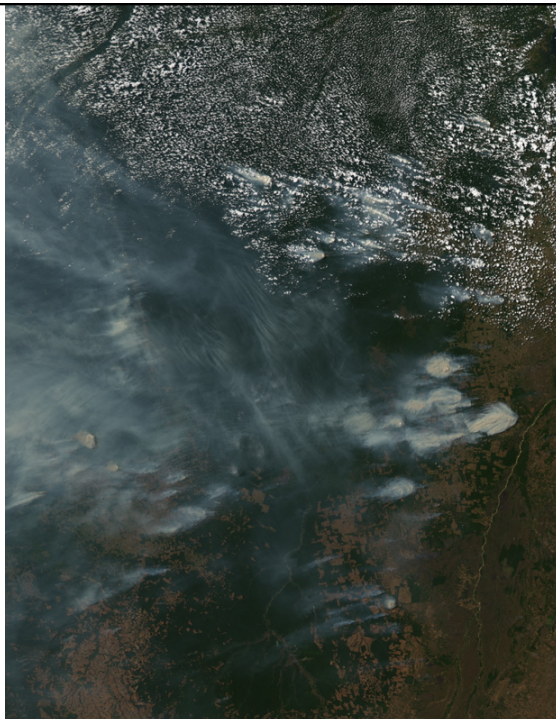
$$\rho_w = 0.1\text{ g m}^{-3}$$

$$\beta_e = 15\text{ km}^{-1}$$



The Aerosol Indirect Effect

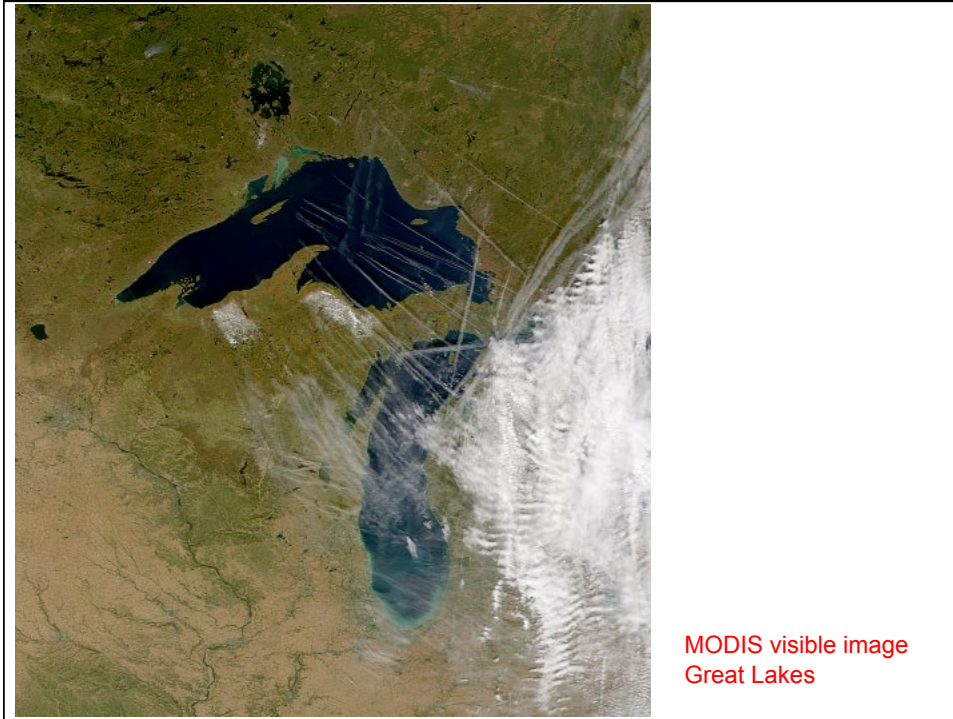
- The impact of aerosols on cloud properties (and hence climate) is called the *aerosol indirect effect*
- A high concentration of aerosols 'overseed' cloud droplets to generate highly concentrated, narrowly distributed cloud droplet spectra
- This can **increase the cloud albedo by up to 30%**, reducing the amount of radiation reaching the surface
- Narrowly distributed cloud droplet spectra **prevent the formation of precipitation** and could increase cloud lifetime that further cools the Earth's surface
- Contrast with the *aerosol direct effect*



Smoke inhibits
cloud formation
over the Amazon

MODIS visible image
Amazon; August 11, 2002

[Koren et al., Science, 2004]



Polydisperse clouds

- The assumption of a single droplet size (*monodisperse*) is not realistic
- Real clouds are *polydisperse* and contain a range of droplet sizes described by a *drop size distribution* $n(r)$

- $n(r) dr$ Number of droplets (per unit volume of air) whose radii fall in the range $[r, r + dr]$

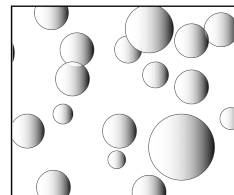
- Dimensions are length⁻⁴; e.g., m⁻³ μm⁻¹

$$N = \int_0^{\infty} n(r) dr \quad \text{Total number density}$$

$$A_{sfc} = \int_0^{\infty} n(r) [4\pi r^2] dr \quad \text{Total surface area}$$

$$\beta_e = \int_0^{\infty} n(r) [Q_e(r) \pi r^2] dr \quad \text{Extinction coefficient}$$

Polydisperse



$$r_{eff} = \frac{\int_0^{\infty} n(r) r^3 dr}{\int_0^{\infty} n(r) r^2 dr}$$

'Effective' radius