Structural Equation Modeling with lavaan

Shane Mueller

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Confirmatory Factor Analysis, Latent Variable Models, and Structural Equation Modeling

Additional Sources:

- http://www.phusewiki.org/docs/Conference%202013%20HE%20Papers/HE06.pdf
- https://blogs.baylor.edu/rlatentvariable/sample-page/r-syntax/

Packages:

- lavaan
- semPlot These install a lot of additional packages for analysis and visualization.

Exploratory factor analysis (EFA) can be used to discover common underlying factors. But sometimes you might have hypotheses about the factor structure, and want to either test or confirm this in your data. The particular factor structure that come out of EFA might be the best bottom-up description, but one associated with a particular theory may be nearly as good, or perhaps even equally good, and it might be better than a description related to an alternate theory. For example, in PCA, we saw a data set created so that factors of visual and verbal intelligence emerged. The solution showed two factors, but one related to general intelligence and one that was a difference between visual and verbal. When we applied exploratory factor analysis, a rotation changed this to map more closely onto our theory, but what if we'd like to identify a particular structure (or set of alternative structures) and determine whether the data are well described by this structure? We can do this with a set of methods known as confirmatory factor analysis, structure equation modeling, and other advanced related methods.

Methods for doing this in the contexct of factor analysis are often called `confirmatory factor analysis`. However, these probably only properly apply to independent factor models. That is, you could create multiple specific models that look like regression equations, but the predictor variable is a latent, hidden variable (much like mixture-modeling). But researchers found that their theoretical models were more complex than this, and the same machinery that permits inferring latent factors can infer much more sit models that determine a more complex structure. There are many varieties of thsee that ego under many n strames; most commonly structure equation models (SEM), latent variable models, path analysis models, and specific applications like latent growth models. Furthermore, advances in hierarchical Bayes models provides substantial ability to infer latent structures. Many of these models (especially the SEM variety) still rely on correlation matrices for the data, and so even those that claim to unearth na causal structure are basing this on correlations, and are really not able to make causal inferences (this relies on experimental design, not statistical inference).

Confirmatory Factor Analysis Example

We will use the lavaan library for this, although other free and commercial projects exist. We will start by looking at the built-in confirmatory factor analysis example in lavaan:

```r
library(lavaan)
# library(semPlot) #this won't compile for me
example(cfa)
```

```
cfa> ## The famous Holzinger and Swineford (1939) example
cfa> HS.model <- ' visual  =~ x1 + x2 + x3
cfa+               textual =~ x4 + x5 + x6
cfa+               speed   =~ x7 + x8 + x9 '

cfa> fit <- cfa(HS.model, data=HolzingerSwineford1939)

cfa> summary(fit, fit.measures=TRUE)
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.896

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3737.745
  Loglikelihood unrestricted model (H1)      -3695.092

  Number of free parameters                         21
  Akaike (AIC)                                7517.490
  Bayesian (BIC)                              7595.339
  Sample-size adjusted Bayesian (BIC)         7528.739

Root Mean Square Error of Approximation:

  RMSEA                                          0.092
  90 Percent Confidence Interval          0.071  0.114
  P-value RMSEA <= 0.05                          0.001

Standardized Root Mean Square Residual:
```

```
  SRMR                                           0.065

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                1.000
    x2                0.554    0.100    5.554    0.000
    x3                0.729    0.109    6.685    0.000
  textual =~
    x4                1.000
    x5                1.113    0.065   17.014    0.000
    x6                0.926    0.055   16.703    0.000
  speed =~
    x7                1.000
    x8                1.180    0.165    7.152    0.000
    x9                1.082    0.151    7.155    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.408    0.074    5.552    0.000
    speed             0.262    0.056    4.660    0.000
  textual ~~
    speed             0.173    0.049    3.518    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.549    0.114    4.833    0.000
   .x2                1.134    0.102   11.146    0.000
   .x3                0.844    0.091    9.317    0.000
   .x4                0.371    0.048    7.779    0.000
   .x5                0.446    0.058    7.642    0.000
   .x6                0.356    0.043    8.277    0.000
   .x7                0.799    0.081    9.823    0.000
   .x8                0.488    0.074    6.573    0.000
   .x9                0.566    0.071    8.003    0.000
    visual            0.809    0.145    5.564    0.000
    textual           0.979    0.112    8.737    0.000
    speed             0.384    0.086    4.451    0.000
```

let's break down this. This uses a data set HolzingerSwineford1939. This data set has nine measures that measure a number of ablities:

- x1: visual perception
- x2: cubes test
- x3: lozenges test
- x4: paragraph comprehension
- x5: sentence completion
- x6: word meaning

- x7: speeded addition
- x8: speeded counting of dots
- x9: speeded discrimination of letters

These have often been thought of having three main components: visual (x1.3), text (x4.6) and speed (x7:9).

```
help(HolzingerSwineford1939)
```

The name lavaan refers to latent variable analysis, which is the essence of confirmatory factor analysis. This is similar to the latent variables we used in mixture modeling (hidden group membership), as well as latent variables used in item response theory.

For simple confirmatory factor analysis, you can think of it as a set of regressions, but instead of an outcome variable, we relate observable measures to a latent variable. For lavaan, we specify a model using a special text markup that isn't exactly R code. Enter the latent variable names on the left, the observed names on the right, separated with =~, and with each factor separated by a line break. Then, you can use the cfa function to fit it using a specified data set. The =~ operator indicates a relationship "is manifasted by", which means that the variable on the right need to actually measured. There are other relationships that you might use in more complex models.

```
HS.model <- " visual   =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 "
fit <- cfa(HS.model, data = HolzingerSwineford1939)
fit
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000
```

```
summary(fit, fit.measures = T)
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000
```

User model versus baseline model:

Comparative Fit Index (CFI)	0.931
Tucker-Lewis Index (TLI)	0.896

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3737.745
Loglikelihood unrestricted model (H1)	-3695.092
Number of free parameters	21
Akaike (AIC)	7517.490
Bayesian (BIC)	7595.339
Sample-size adjusted Bayesian (BIC)	7528.739

Root Mean Square Error of Approximation:

RMSEA		0.092
90 Percent Confidence Interval	0.071	0.114
P-value RMSEA <= 0.05		0.001

Standardized Root Mean Square Residual:

SRMR	0.065

Parameter Estimates:

Information	Expected
Information saturated (h1) model	Structured
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)
visual =~				
x1	1.000			
x2	0.554	0.100	5.554	0.000
x3	0.729	0.109	6.685	0.000
textual =~				
x4	1.000			
x5	1.113	0.065	17.014	0.000
x6	0.926	0.055	16.703	0.000
speed =~				
x7	1.000			
x8	1.180	0.165	7.152	0.000
x9	1.082	0.151	7.155	0.000

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
visual ~~				
textual	0.408	0.074	5.552	0.000
speed	0.262	0.056	4.660	0.000
textual ~~				
speed	0.173	0.049	3.518	0.000

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.549    0.114    4.833    0.000
   .x2                1.134    0.102   11.146    0.000
   .x3                0.844    0.091    9.317    0.000
   .x4                0.371    0.048    7.779    0.000
   .x5                0.446    0.058    7.642    0.000
   .x6                0.356    0.043    8.277    0.000
   .x7                0.799    0.081    9.823    0.000
   .x8                0.488    0.074    6.573    0.000
   .x9                0.566    0.071    8.003    0.000
    visual            0.809    0.145    5.564    0.000
    textual           0.979    0.112    8.737    0.000
    speed             0.384    0.086    4.451    0.000
```

The above model shoul produce the same model that the 'example' did.

```
HS.model2 <- " visual   =~ x1 + x2 + x3 + x4 + x5 + x6
               speed    =~ x7 + x8 + x9 "
fit2 <- cfa(HS.model2, data = HolzingerSwineford1939)
fit2
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         19

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                     181.337
  Degrees of freedom                                26
  P-value (Chi-square)                           0.000
```

```
summary(fit2, fit.measures = T)
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         19

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                     181.337
  Degrees of freedom                                26
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:
```

```
  Comparative Fit Index (CFI)                    0.824
  Tucker-Lewis Index (TLI)                       0.756

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3785.760
  Loglikelihood unrestricted model (H1)      -3695.092

  Number of free parameters                         19
  Akaike (AIC)                                7609.521
  Bayesian (BIC)                              7679.956
  Sample-size adjusted Bayesian (BIC)         7619.699

Root Mean Square Error of Approximation:

  RMSEA                                          0.141
  90 Percent Confidence Interval          0.122  0.161
  P-value RMSEA <= 0.05                          0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.113

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                1.000
    x2                0.509    0.156    3.254    0.001
    x3                0.477    0.150    3.189    0.001
    x4                1.992    0.272    7.314    0.000
    x5                2.194    0.300    7.303    0.000
    x6                1.852    0.254    7.294    0.000
  speed =~
    x7                1.000
    x8                1.148    0.163    7.054    0.000
    x9                0.891    0.124    7.189    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    speed             0.095    0.029    3.287    0.001

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                1.112    0.093   11.922    0.000
   .x2                1.318    0.108   12.193    0.000
   .x3                1.219    0.100   12.196    0.000
```

```
   .x4               0.373    0.047    7.853    0.000
   .x5               0.474    0.059    8.059    0.000
   .x6               0.351    0.043    8.221    0.000
   .x7               0.732    0.083    8.821    0.000
   .x8               0.428    0.082    5.201    0.000
   .x9               0.658    0.071    9.300    0.000
    visual           0.246    0.067    3.661    0.000
    speed            0.451    0.095    4.733    0.000
```

```
partable(fit)
```

```
   id     lhs op     rhs user block group free ustart exo label plabel
1   1  visual =~      x1    1     1     1    0      1   0         .p1.
2   2  visual =~      x2    1     1     1    1     NA   0         .p2.
3   3  visual =~      x3    1     1     1    2     NA   0         .p3.
4   4 textual =~      x4    1     1     1    0      1   0         .p4.
5   5 textual =~      x5    1     1     1    3     NA   0         .p5.
   start   est    se
1  1.000 1.000 0.000
2  0.778 0.554 0.100
3  1.107 0.729 0.109
4  1.000 1.000 0.000
5  1.133 1.113 0.065
 [ reached getOption("max.print") -- omitted 19 rows ]
```

```
# install.packages('semPlot',dependencies=T)
library(psych)
## library(semPlot) ##this won't install for me! semPaths(fit,
## 'model','est',curvePivot = FALSE,edge.label.cex = 0.5)

lavaan.diagram(fit, cut = 0.2, digits = 2)  ##in psych package
summary(fit, fit.measures = T, standardize = T)
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      85.306
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.896
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3737.745
  Loglikelihood unrestricted model (H1)      -3695.092

  Number of free parameters                         21
  Akaike (AIC)                                7517.490
  Bayesian (BIC)                              7595.339
  Sample-size adjusted Bayesian (BIC)         7528.739

Root Mean Square Error of Approximation:

  RMSEA                                          0.092
  90 Percent Confidence Interval          0.071  0.114
  P-value RMSEA <= 0.05                          0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.065

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  visual =~
    x1                1.000                               0.900    0.772
    x2                0.554    0.100    5.554    0.000    0.498    0.424
    x3                0.729    0.109    6.685    0.000    0.656    0.581
  textual =~
    x4                1.000                               0.990    0.852
    x5                1.113    0.065   17.014    0.000    1.102    0.855
    x6                0.926    0.055   16.703    0.000    0.917    0.838
  speed =~
    x7                1.000                               0.619    0.570
 [ reached getOption("max.print") -- omitted 2 rows ]

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  visual ~~
    textual           0.408    0.074    5.552    0.000    0.459    0.459
    speed             0.262    0.056    4.660    0.000    0.471    0.471
  textual ~~
    speed             0.173    0.049    3.518    0.000    0.283    0.283

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .x1                0.549    0.114    4.833    0.000    0.549    0.404
   .x2                1.134    0.102   11.146    0.000    1.134    0.821
   .x3                0.844    0.091    9.317    0.000    0.844    0.662
```

```
   .x4                0.371    0.048    7.779    0.000    0.371    0.275
   .x5                0.446    0.058    7.642    0.000    0.446    0.269
   .x6                0.356    0.043    8.277    0.000    0.356    0.298
   .x7                0.799    0.081    9.823    0.000    0.799    0.676
   .x8                0.488    0.074    6.573    0.000    0.488    0.477
   .x9                0.566    0.071    8.003    0.000    0.566    0.558
    visual            0.809    0.145    5.564    0.000    1.000    1.000
 [ reached getOption("max.print") -- omitted 2 rows ]
```

```
inspect(fit, "rsquare")
```

```
   x1    x2    x3    x4    x5    x6    x7    x8    x9
0.596 0.179 0.338 0.725 0.731 0.702 0.324 0.523 0.442
```

This gives some tests for whether the models are any good. First, let's look at the diagram.

The circles indicate latent variables. The boxes indicate measured variables. two-way arrows indicate covariances; one-way arrows indicate a scaled covariance between latentt and measured variables (the ~= operation), and this can be interpreted as a factor loading much like exploratory factor analysis.

Notice that for each latent variable, one loading is a dashed line, and 'fixed' at 1.0. This is done because there is a lack of identifiability for latent variables–you don't really know what values they take. So by default, they are pinned to the first measure, and other loadings are expressed relative to that. So, x1 is the strongest contributor to visual performance; x5 the strongest to text, and x8 the strongest to speed. Connection that point in a circle to the same node indicate variances, and can be used to assess reliability of estimates. Finally, two-way arrows indicate covariances between latent variables. Here, we see that the relationships between the three latent variables is small but positive–with scaled covariances of .41, .26, and .17. These numbers appear in the summary() of the fitted model as well.

The summary function provides many statistics that will help interpretation. David Kenny's page has more details on some of these that were not in standard factor analysis http://davidakenny.net/cm/fit.htm

CFI (comparative fit index) is like TLI, but will always be higher as it does not penalize complexity as much. Kenny says CFI/TLI are not meaningful unless RMSEA is below about .15. In this case RMSE is .09, which is OK but not great.

Testing alternate models

The model is reasonable, but the more powerful argument is to compare it to other models that are also reasonable. Vsl and text are correlated; maybe they really don't belong as separate latent factors.

```
HS.model2 <- " vt  =~ x1 + x2 + x3 + x4 + x5 + x6
               speed   =~ x7 + x8 + x9 "
fit2 <- cfa(HS.model2, data = HolzingerSwineford1939)
fit2
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         19

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                     181.337
  Degrees of freedom                                26
  P-value (Chi-square)                           0.000
```

These are both 'significant' but the test statistic differs a lot. We can test whether they differ significantly using the anova function, which also gives us AIC/BIC statistics.

```
anova(fit, fit2)
```

```
Chi Square Difference Test

     Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
fit  24 7517.5 7595.3  85.305
fit2 26 7609.5 7680.0 181.337     96.031       2  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(fit, fit.measures = T)
```

```
lavaan 0.6-3 ended normally after 35 iterations

  Optimization method                           NLMINB
  Number of free parameters                         21

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  Estimator                                         ML
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Standardized Root Mean Square Residual:
```

```
  SRMR                                           0.065

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  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
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   .x5                0.446    0.058    7.642    0.000
   .x6                0.356    0.043    8.277    0.000
   .x7                0.799    0.081    9.823    0.000
   .x8                0.488    0.074    6.573    0.000
   .x9                0.566    0.071    8.003    0.000
    visual            0.809    0.145    5.564    0.000
    textual           0.979    0.112    8.737    0.000
    speed             0.384    0.086    4.451    0.000
```

```
summary(fit2, fit.measures = T)
```

```
lavaan 0.6-3 ended normally after 35 iterations

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```

```
  Estimator                                         ML
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  Sample-size adjusted Bayesian (BIC)         7619.699

Root Mean Square Error of Approximation:

  RMSEA                                         0.141
  90 Percent Confidence Interval          0.122  0.161
  P-value RMSEA <= 0.05                         0.000

Standardized Root Mean Square Residual:

  SRMR                                          0.113

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  vt =~
    x1                1.000
    x2                0.509    0.156    3.254    0.001
    x3                0.477    0.150    3.189    0.001
    x4                1.992    0.272    7.314    0.000
    x5                2.194    0.300    7.303    0.000
    x6                1.852    0.254    7.294    0.000
  speed =~
    x7                1.000
```

```
    x8                1.148    0.163    7.054    0.000
    x9                0.891    0.124    7.189    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  vt ~~
    speed             0.095    0.029    3.287    0.001

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                1.112    0.093   11.922    0.000
   .x2                1.318    0.108   12.193    0.000
   .x3                1.219    0.100   12.196    0.000
   .x4                0.373    0.047    7.853    0.000
   .x5                0.474    0.059    8.059    0.000
   .x6                0.351    0.043    8.221    0.000
   .x7                0.732    0.083    8.821    0.000
   .x8                0.428    0.082    5.201    0.000
   .x9                0.658    0.071    9.300    0.000
    vt                0.246    0.067    3.661    0.000
    speed             0.451    0.095    4.733    0.000
```

```
diagram(fit)
diagram(fit2)
```

This suggests that the smaller model is fit more poorly according to RMSEA and TLI measures. Also, BIC is 7620 for the smaller model vs 7628 for the larger model, as are the AIC and related measures. Finally, anova chi-squared test is significantly different, indicating the larger model (fit) accounts for the data better than the smaller model (fit2). Altogether, this justifies the 3-factor model over a 2-factor model.

What if we give it a model we don't think is correct–here f2 is cobbled together:

```
HS.model3 <- " f1  =~ x1 + x2
               f2 =~ x3 + x4
               f3 =~ x5 + x6
               f4   =~ x7 + x8 + x9 "
fit3 <- cfa(HS.model3, data = HolzingerSwineford1939)
fit3
```

```
lavaan 0.6-3 ended normally after 60 iterations

  Optimization method                           NLMINB
  Number of free parameters                         24

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                     144.108
  Degrees of freedom                                21
  P-value (Chi-square)                           0.000
```

There is a warning, but it does produce a result. We can use an anova function to compare the two:

```
anova(fit, fit3)
```

```
Chi Square Difference Test
```

```
     Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
fit3 21 7582.3 7671.3 144.108
fit  24 7517.5 7595.3  85.305    -58.803       3          1
```

```
# semPlot::semPaths(fit3,title=F,curvePilot=T)
lavaan.diagram(fit3, cut = 0.2, digits = 2)
```

Now, there new more complex model, which has more parameters and fewer DF,fits more poorly than the smaller model, so we would again prefer the 3-parameter model.

The Bifactor model

The bifactor model describes a model where everything is related to one general factor (g), but there are several other specific factors other things are related to.

```
HS.model4 <- " g =~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9
               f1  =~ x1 + x2 + x3
               f2 =~ x4 + x5 + x6
               f3   =~ x7 + x8 + x9
               g ~~ 0*f1
               g ~~ 0*f2
               g ~~ 0*f3
               f1~~ 0*f1
               f2~~ 0*f2
               f3 ~~ 0*f3
"


fit4 <- cfa(HS.model4, data = HolzingerSwineford1939)
# fit4 <- cfa(HS.model4,data=HolzingerSwineford1939,estimator='WLS') fit4
summary(fit4, fit.measures = T)
```

```
lavaan 0.6-3 ended normally after 48 iterations

  Optimization method                           NLMINB
  Number of free parameters                         27

  Number of observations                           301

  Estimator                                         ML
  Model Fit Test Statistic                      56.159
  Degrees of freedom                                18
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic              918.852
  Degrees of freedom                                36
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.957
  Tucker-Lewis Index (TLI)                       0.914
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3723.172
  Loglikelihood unrestricted model (H1)      -3695.092

  Number of free parameters                         27
  Akaike (AIC)                                7500.344
  Bayesian (BIC)                              7600.435
  Sample-size adjusted Bayesian (BIC)         7514.807

Root Mean Square Error of Approximation:

  RMSEA                                          0.084
  90 Percent Confidence Interval          0.060  0.109
  P-value RMSEA <= 0.05                          0.013

Standardized Root Mean Square Residual:

  SRMR                                           0.051

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  g =~
    x1                1.000
    x2                0.685    0.121    5.655    0.000
    x3                0.921    0.151    6.118    0.000
    x4                1.235    0.144    8.579    0.000
    x5                1.401    0.163    8.623    0.000
    x6                1.140    0.134    8.536    0.000
    x7                0.841    0.125    6.746    0.000
    x8                0.986    0.133    7.427    0.000
    x9                0.725    0.111    6.557    0.000
  f1 =~
    x1                1.000
    x2                0.893    0.238    3.758    0.000
    x3                1.205    0.299    4.033    0.000
  f2 =~
 [ reached getOption("max.print") -- omitted 7 rows ]

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  g ~~
    f1                0.000
    f2                0.000
    f3                0.000
  f1 ~~
    f2               -0.411    0.084   -4.897    0.000
```

```
    f3                -0.349    0.081   -4.320    0.000
  f2 ~~
    f3                -0.523    0.084   -6.225    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    f1                0.000
    f2                0.000
    f3                0.000
   .x1                0.703    0.113    6.252    0.000
   .x2                1.070    0.101   10.545    0.000
   .x3                0.711    0.102    6.996    0.000
   .x4                0.383    0.048    7.939    0.000
   .x5                0.416    0.058    7.143    0.000
   .x6                0.370    0.044    8.482    0.000
   .x7                0.737    0.084    8.751    0.000
   .x8                0.406    0.086    4.704    0.000
   .x9                0.680    0.071    9.558    0.000
    g                 0.640    0.135    4.748    0.000
```

```
# semPaths(fit4,'model','est',curvePivot = FALSE,edge.label.cex = 0.5)
# semPaths(fit4,'model','est',layout='circle',curvePivot =
# FALSE,edge.label.cex = 0.5)
# semPaths(fit4,'model','est',layout='tree',curvePivot =
# FALSE,edge.label.cex = 0.5)
lavaan.diagram(fit4, cut = 0.2, digits = 2)

anova(fit4, fit)
```

```
Chi Square Difference Test

     Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit4 18 7500.3 7600.4 56.159
fit  24 7517.5 7595.3 85.305     29.146       6  5.708e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here, we have mixed results; a Chi-quared test shows the more complex model is better, as does AIC; but BIC is pessimistic, feeling it is too complex. Looking at the parameter estimates, there are some strange things; we no longer have consistent positive loadings to f1, and we get a warning that it is not positive definite, which should make us wary that our data is not sufficiently constraining our model.

We can force the latent variables to be uncorrelated. In this case, the model does not converge, and so there is not much we can take away from it, but it might be useful in some situations. Plus, below we can see code that let's us see the residuals of the correlation matrix, the basis for RMSE and RMSEA measures.

```
fit5 <- cfa(HS.model4, data = HolzingerSwineford1939, orthogonal = TRUE)
summary(fit5)
```

```
lavaan 0.6-3 ended normally after 31 iterations

  Optimization method                           NLMINB
  Number of free parameters                         24

  Number of observations                           301
```

```
  Estimator                                         ML
  Model Fit Test Statistic                     312.264
  Degrees of freedom                            21
  P-value (Chi-square)                          0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  g =~
    x1                1.000
    x2                0.508       NA
    x3                0.493       NA
    x4                1.930       NA
    x5                2.123       NA
    x6                1.796       NA
    x7                0.385       NA
    x8                0.398       NA
    x9                0.606       NA
  f1 =~
    x1                1.000
    x2                0.778       NA
    x3                1.107       NA
  f2 =~
 [ reached getOption("max.print") -- omitted 7 rows ]

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  g ~~
    f1                0.000
    f2                0.000
    f3                0.000
  f1 ~~
    f2                0.000
    f3                0.000
  f2 ~~
    f3                0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    f1                0.000
    f2                0.000
    f3                0.000
   .x1                1.098       NA
   .x2                1.315       NA
   .x3                1.212       NA
   .x4                0.380       NA
   .x5                0.486       NA
   .x6                0.356       NA
   .x7                1.145       NA
```

```
   .x8                0.981       NA
   .x9                0.919       NA
    g                 0.261       NA
```

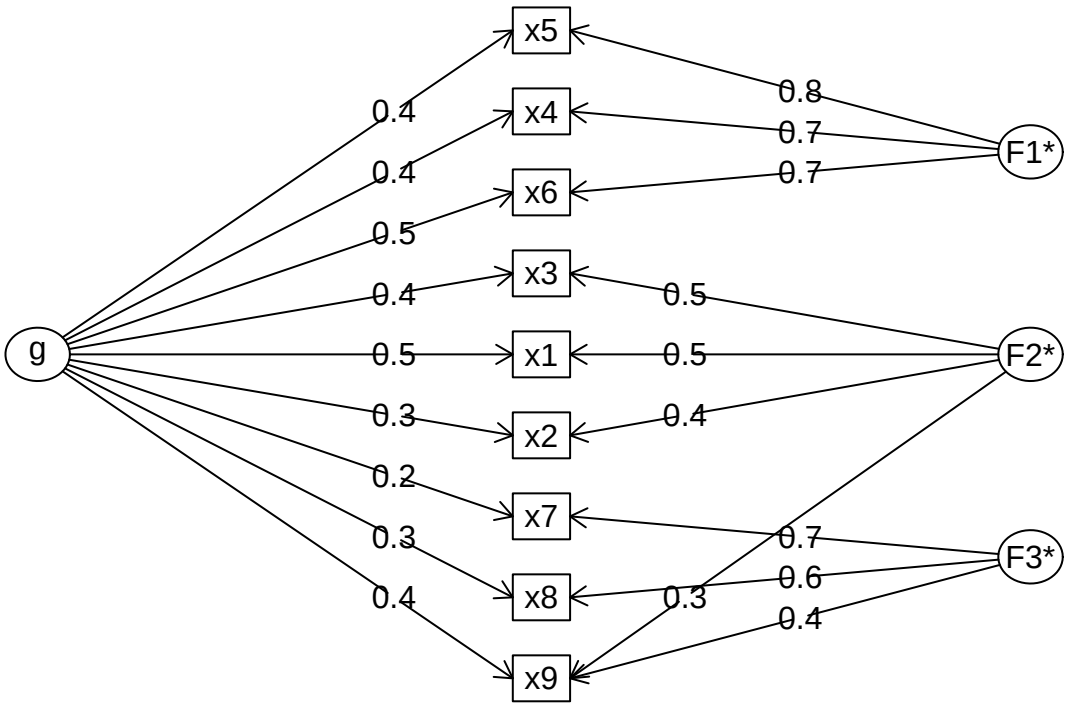
```
# semPaths(fit5,'model','est',curvePivot = FALSE,edge.label.cex = 0.5)
residuals(fit5, type = "cor")
```

```
$type
[1] "cor.bollen"

$cov
   x1     x2     x3     x4     x5     x6     x7     x8     x9
x1  0.000
x2  0.201  0.000
x3  0.343  0.291  0.000
x4  0.001 -0.034 -0.030  0.000
x5 -0.075 -0.046 -0.110  0.020  0.000
x6 -0.010  0.008  0.011 -0.006  0.015  0.000
x7 -0.012 -0.115  0.032  0.021 -0.050 -0.030  0.000
x8  0.136  0.048  0.141 -0.064 -0.031 -0.019  0.450  0.000
 [ reached getOption("max.print") -- omitted 1 row ]
```

Bifactor model with the omega function

A traditional version of the bifactor model is available in the omega function, where the factors are automatically identified. This is more like EFA with a specific structure.

```
library(psych)
holz <- omega(cor(HolzingerSwineford1939[, 7:15]), 3, title = "14 ability tests from Holzinger-Swinefor
```




```
holz
```

```
14 ability tests from Holzinger-Swineford
Call: omega(m = cor(HolzingerSwineford1939[, 7:15]), nfactors = 3,
    title = "14 ability tests from Holzinger-Swineford")
Alpha:                 0.76
G.6:                   0.81
Omega Hierarchical:    0.45
Omega H asymptotic:    0.53
Omega Total            0.85

Schmid Leiman Factor loadings greater than  0.2
      g   F1*   F2*   F3*   h2   u2   p2
x1 0.49        0.46       0.48 0.52 0.49
x2 0.29        0.40       0.26 0.74 0.33
x3 0.41        0.53       0.45 0.55 0.37
x4 0.45  0.73             0.73 0.27 0.28
x5 0.42  0.76             0.75 0.25 0.23
x6 0.46  0.69             0.69 0.31 0.30
x7 0.23              0.67 0.52 0.48 0.10
x8 0.35              0.62 0.52 0.48 0.23
x9 0.45        0.30  0.42 0.46 0.54 0.43

With eigenvalues of:
   g  F1*  F2*  F3*
1.44 1.62 0.77 1.03

general/max  0.89   max/min =   2.09
```

```
mean percent general =  0.31    with sd =  0.12 and cv of  0.38
Explained Common Variance of the general factor =  0.3

The degrees of freedom are 12  and the fit is  0.08

The root mean square of the residuals is  0.02
The df corrected root mean square of the residuals is  0.03

Compare this with the adequacy of just a general factor and no group factors
The degrees of freedom for just the general factor are 27  and the fit is  1.75

The root mean square of the residuals is  0.2
The df corrected root mean square of the residuals is  0.23

Measures of factor score adequacy
                                                 g  F1*   F2*  F3*
Correlation of scores with factors             0.68 0.84  0.67 0.79
Multiple R square of scores with factors       0.46 0.71  0.45 0.62
Minimum correlation of factor score estimates -0.08 0.42 -0.10 0.23

 Total, General and Subset omega for each subset
                                                 g  F1*  F2*  F3*
Omega total for total scores and subscales    0.85 0.89 0.65 0.72
Omega general for total scores and subscales  0.45 0.24 0.27 0.19
Omega group for total scores and subscales    0.35 0.65 0.37 0.53
```

More complex models

Confirmatory factor analysis typically identifies a single set of factors and tries to model the data in that way. But the lavaan library offers more complex structural equation modeling and latent growth curve modeling, and general latent variable regressions, which is also useful in complex situations. For example, you could combine several measures to create a factor, which you then use as a regression predictor or outcome variable to establish relationships between hidden/latent variables. In developmental circles, it is common to use latent growth models to model the development or decline of specific latent cognitive or social behaviors or skills.

A common type of analysis that lavaan permits is looking at the role of mediating variables. There are many tools available for specifically looking at 3-variable problems, but lavaan lets you model arbitrarily complex mediation schemes. The analysis of these is more ad hoc though.

Mediation Analysis

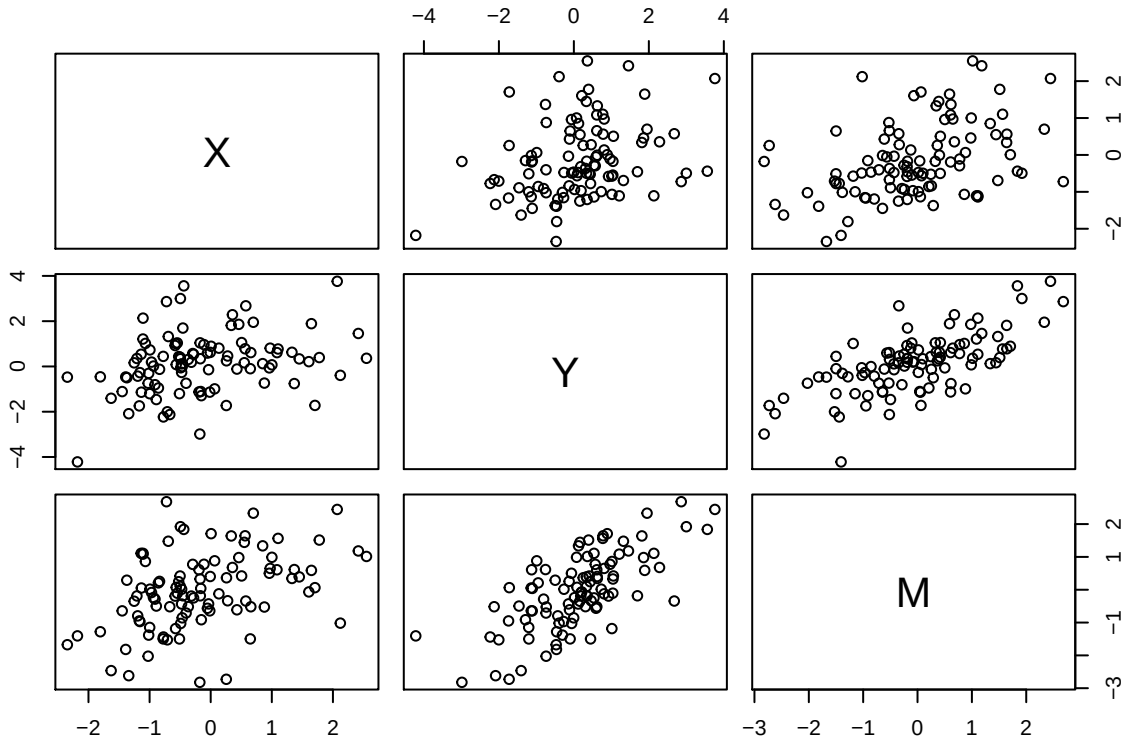
The notion of doing a mediation analysis is central to many areas of psychology, especially in social domains where you cannot have complete experimental control. In these cases, you will always have potentially confounding variables, and mediation analysis examines a particular kind of causal relationship. Some of the papers describing mediation are among the top 20 scientific research papers cited in any domain.

The notion is that you may observe a correlation between two variables, but also observe that these may be correlated with a third. For example, you might find that as a population ages from teenager to adult, vehicular accidents go down. But, it might not be age per se that causes this. Instead, it might be practice... as you get older, you get more experience driving, and the experience reduces accidents. Experience may be a mediating variable here. Can all the improvement in driver safety be attributed to experience? Maybe, but

to find out, you need to study people who started driving at different ages. This would be a classic mediation test.

Below is an example of how we would model this with lavaan. Here, using standard terminology, y is the outcome (accident rate), x is the predictor (age), and m is the mediator variable (experience).

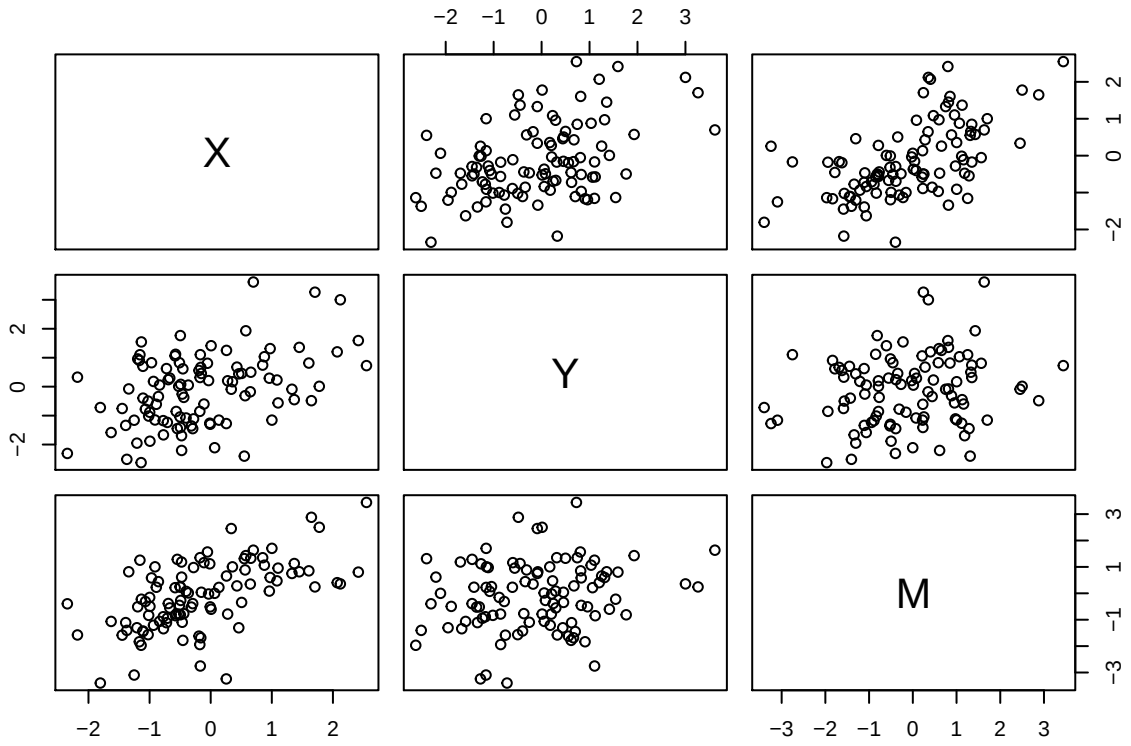
```
set.seed(1234)

X <- rnorm(100)
M <- 0.5 * X + rnorm(100)
Y <- 0.7 * M + rnorm(100)

M2 <- 0.6 * X + rnorm(100)
Y2 <- 0.6 * X + rnorm(100)


Data <- data.frame(X = X, Y = Y, M = M)
Data2 <- data.frame(X = X, Y = Y2, M = M2)

pairs(Data)
```




```
cor(Data)
```

```
          X         Y         M
X 1.0000000 0.3122748 0.4188856
Y 0.3122748 1.0000000 0.6909791
M 0.4188856 0.6909791 1.0000000
```

```
pairs(Data2)
```




```
cor(Data2)
```

```
          X         Y         M
X 1.0000000 0.4255321 0.5857204
Y 0.4255321 1.0000000 0.1827901
M 0.5857204 0.1827901 1.0000000
```

```
model <- " # direct effect
Y ~ c*X
# mediator
M ~ a*X
Y ~ b*M
# indirect effect (a*b)
ab := a*b
# total effect
total := c + (a*b)
"

library(lavaan)
fit <- sem(model, data = Data)
summary(fit)
```

```
lavaan 0.6-3 ended normally after 12 iterations

  Optimization method                           NLMINB
  Number of free parameters                          5

  Number of observations                           100

  Estimator                                         ML
  Model Fit Test Statistic                       0.000
```

```
  Degrees of freedom                                 0
  Minimum Function Value               0.0000000000000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X          (c)    0.036    0.104    0.348    0.728
  M ~
    X          (a)    0.474    0.103    4.613    0.000
  Y ~
    M          (b)    0.788    0.092    8.539    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Y                 0.898    0.127    7.071    0.000
   .M                 1.054    0.149    7.071    0.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab                0.374    0.092    4.059    0.000
    total             0.410    0.125    3.287    0.001
```

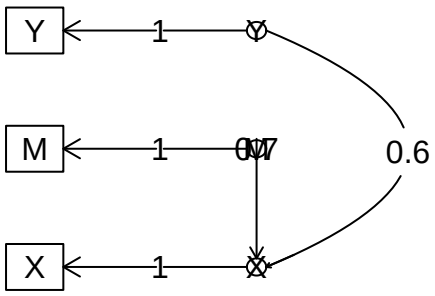
```
fit2 <- sem(model, data = Data2)
summary(fit2)
```

```
lavaan 0.6-3 ended normally after 15 iterations

  Optimization method                           NLMINB
  Number of free parameters                          5

  Number of observations                           100

  Estimator                                         ML
  Model Fit Test Statistic                       0.000
  Degrees of freedom                                 0

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X          (c)    0.594    0.136    4.360    0.000
  M ~
    X          (a)    0.748    0.104    7.227    0.000
  Y ~
```

```
    M          (b)   -0.097    0.107   -0.910    0.363

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Y                 1.218    0.172    7.071    0.000
   .M                 1.070    0.151    7.071    0.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab               -0.073    0.080   -0.903    0.367
    total             0.521    0.111    4.702    0.000
```

```
# library(semPlot)
lavaan.diagram(fit2)
```

Structural model




```
# semPaths(fit2,title=F,curvePilot=T)
```