Finding all EQUAL PAIRS in Two Sorted Arrays

> *It takes a really bad school to ruin a good student*
> *and*
> *a really fantastic school to rescue a bad student.*
>
> ***Dennis J. Frailey***

Definition: Equal Pair

Given two sets of data **X** $= \{ x_1, x_2, \ldots, x_m\}$ and **Y** $= \{ y_1, y_2, \ldots, y_n\}$, an **equal pair** is an element x_i in **X** and an element y_j in **Y** such that $x_i = y_j$.

Problem Statement

Given two arrays `x[ ]` and `y[ ]`, each of which contains **sorted** and **distinct** integers in **ascending** order, write a **C** program to report all equal pairs.

First Thought: 1/5

```
int x[m], y[n];

for (i = 0; i < m; i++) {
    for (j = 0; j < n; j++) {
        if (x[i] == y[j]) {
            // an equal pair found
            // report it
        }
        else {
            // not an equal pair
            // perhaps do nothing
        }
    }
}
```

1. Suppose arrays `x[]` and `y[]` have `m` and `n` elements.
2. The first thought may be this:
   a) For each `i`, compare `x[i]` with every `y[j]`.
   b) Report any found equal pairs.

First Thought: 2/5

```
int x[m], y[n];

for (i = 0; i < m; i++) {
    for (j = 0; j < n; j++) {
        if (x[i] == y[j]) {
            // an equal pair found
            // report it
        }
        else {
            // not an equal pair
            // perhaps do nothing
        }
    }
}
```

1. If `x[i]` and `y[j]` are not an equal pair, obviously nothing should happen.
2. The `else` part is empty and we should move on to the next iteration, comparing `x[i]` and `y[j+1]` .

First Thought: 3/5

```
int x[m], y[n];

for (i = 0; i < m; i++) {
   for (j = 0; j < n; j++) {
      if (x[i] == y[j]) {
         // an equal pair found
         // report it
      }
      else {
         // not an equal pair
         // perhaps do nothing
      }
   }
}
```

1. If `x[i]` and `y[j]` are equal, we should record this.
2. The `then` part may have to store `i` and `j` and increase the "count" by 1 before moving on to comparing `x[i]` and `y[j+1]`.

First Thought: 4/5

```
int x[m], y[n];
int xx[ ], yy[ ], k = 0;

for (i = 0; i < m; i++)
   for (j = 0; j < n; j++) {
      if (x[i] == y[j]) {
         xx[k] = i;
         yy[k] = j;
         k++;
         break;
      }
}
```

Because the array elements are distinct, once a pair is found, `y[j+1]` is a different number.

1. We need arrays `xx[ ]` and `yy[ ]` for saving the positions of the found equal pairs.
2. The `int k` is a counter to keep track of the number of equal pairs found so far.
3. The `break` is used to get out of the `j`-loop as once an equal pair is found, we could skip the remaining of the `j`-loop.
4. **Why?** The arrays have **distinct** numbers.

First Thought: 5/5

```
int EQUAL_PAIRS(int x[], int y[],
          int m, int n,
          int xx[], int yy[])
{
     int i, j, k;

     k = 0;
     for (i = 0; i < m; i++)
          for (j = 0; j < n; j++)
               if (x[i] == y[j]) {
                    xx[k] = i;
                    yy[k] = j;
                    k++;
                    break;
               }
     return k;
}
```

1. Let us write a function `EQUAL_PAIRS()` for this problem:
   a) `int x[ ]` and `y[ ]` are the input arrays with `m` and `n` elements.
   b) `xx[ ]` and `yy[ ]` are the returned arrays that store the locations.
   c) `k`, the function value, is the number of equal pairs.

Is This Good?: 1/2

```
int EQUAL_PAIRS(int x[], int y[],
          int m, int n,
          int xx[], int yy[])
{
     int i, j, k;

     k = 0;
     for (i = 0; i < m; i++)
          for (j = 0; j < n; j++)
               if (x[i] == y[j]) {
                    xx[k] = i;
                    yy[k] = j;
                    k++;
                    break;
               }
     return k;
}
```

1. Let us determine the number of comparisons used.
2. In the worst case, if `x[i]` is not equal to any element of array `y[ ]`, `n` comparisons are needed for the `j`-loop.
3. The `i`-loop iterates `m` times, and, in the worst case, the total number of comparisons is $O(m \times n)$, not very good.
4. If $m=n$, it is an $O(n^2)$ solution.

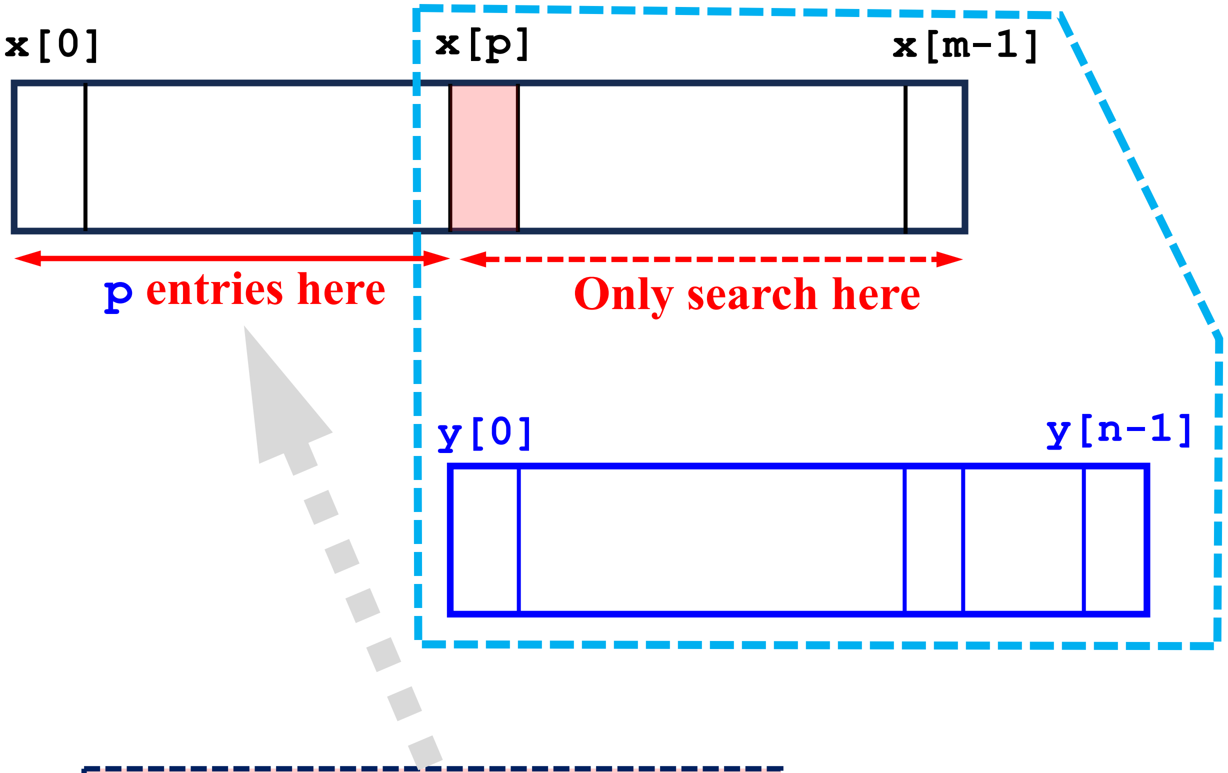
Is This Good?: 2/2

```
int EQUAL_PAIRS(int x[], int y[],
          int m, int n,
          int xx[], int yy[])
{
     int i, j, k;

     k = 0;
     for (i = 0; i < m; i++)
          for (j = 0; j < n; j++)
               if (x[i] == y[j]) {
                    xx[k] = i;
                    yy[k] = j;
                    k++;
                    break;
               }
     return k;
}
```

1. This could be an acceptable solution for beginners.
2. However, it does not use all of the given conditions such as **sorted** to its maximum.
3. It is easy to improve this version to some degree; but these could just be minor.
4. Well, let us see how we could improve this version for beginners.

Improvements? 1/5

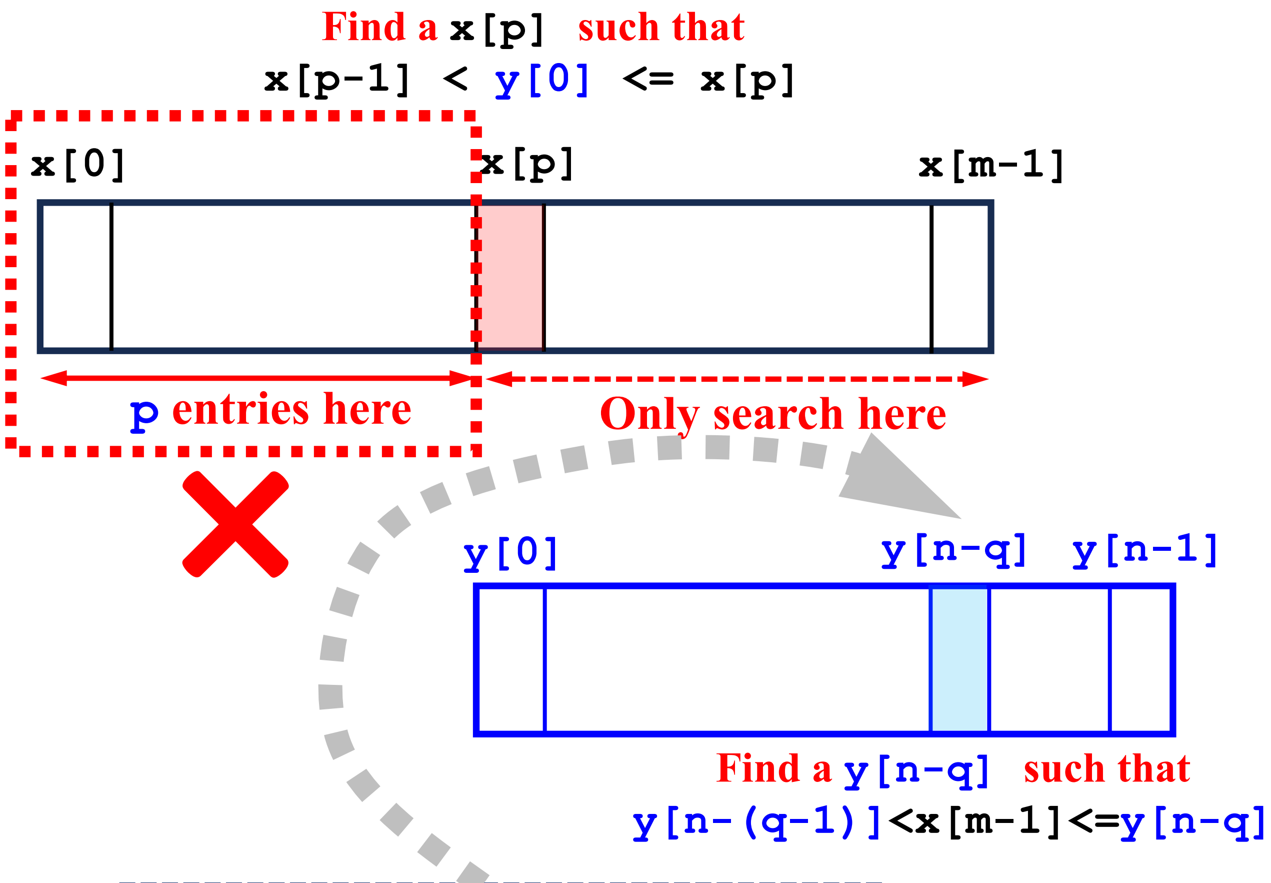
Find a x[p] such that
x[p-1] < y[0] <= x[p]



```
for (i = 0; i < m; i++)
   if (x[i] >= y[0]) {
      p = i;
      break;
   }
```

1. Let us assume x[0] < y[0].
2. Find a x[p] such that
   x[p-1] < y[0] <= x[p]
4. In this way, we do not compare x[0] to x[p-1] because they cannot be found in x[].
5. Hence, we have to handle x[p..m-1] and y[0..n-1] using p comparisons.
6. Can we do more? **YES!**

Improvements? 2/5

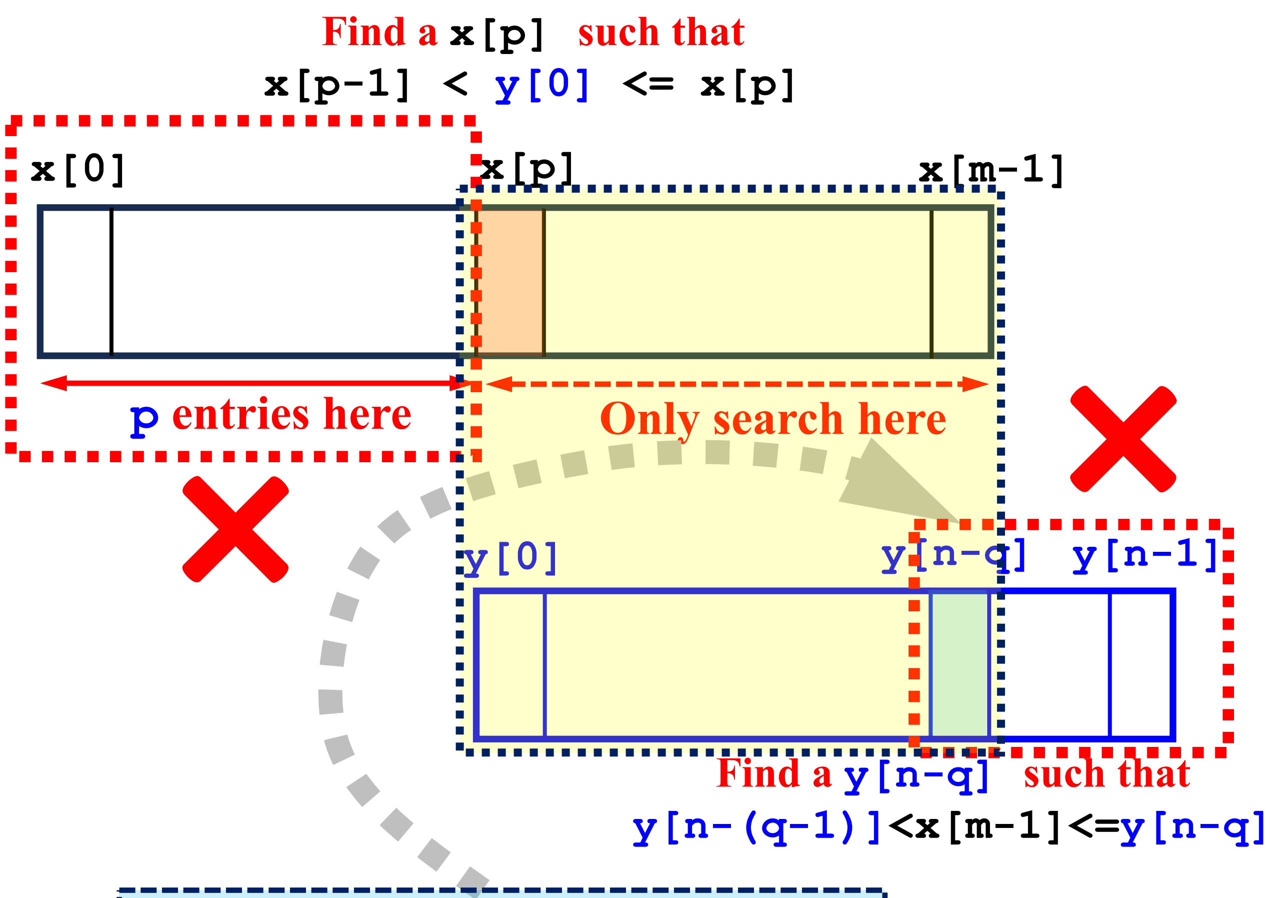


```
for (j = n-1; j >= 0; j--)
   if (y[j] < x[m-1]) {
      q = n-j-1;
      break;
   }
```

1. If `y[n-1] > x[m-1]`, we could cut the comparison range further.
2. Find a `y[n-q]` such that

   `y[n-(q-1)] < x[m-1] <= y[n-q]`
3. In this way, from `y[n-q]` to `y[n-1]` cannot be in `x[]` because they are all larger than the entries in `x[]`.
4. The ranges are reduced to `x[p..m-1]` and `y[0..n-q]`.

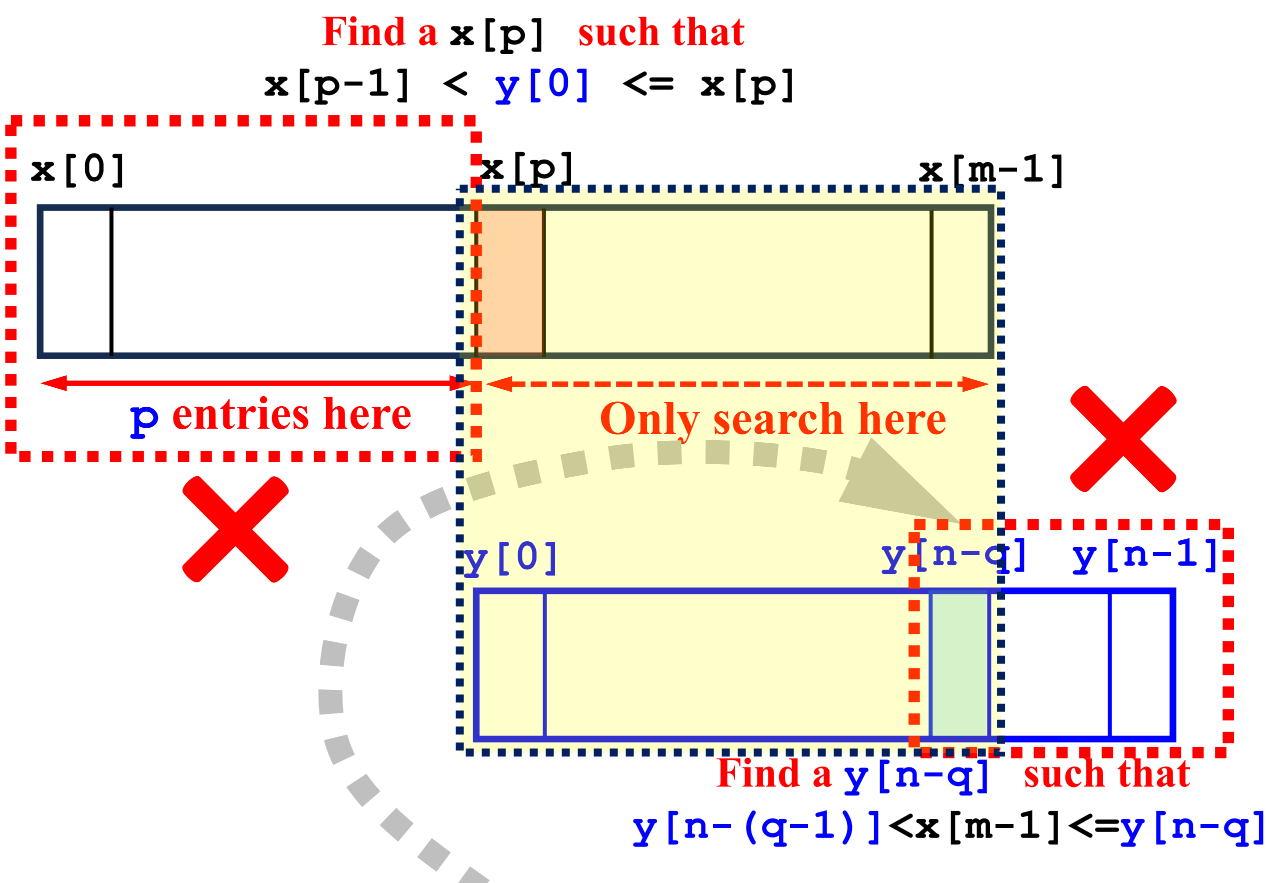
Improvements? 3/5



```
for (j = n-1; j >= 0; j--)
   if (y[j] < x[m-1]) {
      q = n-j-1;
      break;
   }
```

1. Thus, **`p`** comparisons are used to cut the first **`p`** elements from **`x[]`** and **`q`** comparisons to cut **`q`** elements from **`y[]`**.
2. The remaining elements in **`x[]`** is **`m-p`** and the remaining elements in **`y[]`** is **`n-q`**.
3. Therefore, each **`x[i]`** in the **`x[]`** is compared with every entries in **`y[0..(n-q)]`** which requires *n-q* comparisons.

Improvements? 4/5



Find a x[p] such that

x[p-1] < y[0] <= x[p]

x[0]

x[p]

x[m-1]

p entries here

Only search here

y[0]

y[n-q]

y[n-1]

Find a y[n-q] such that

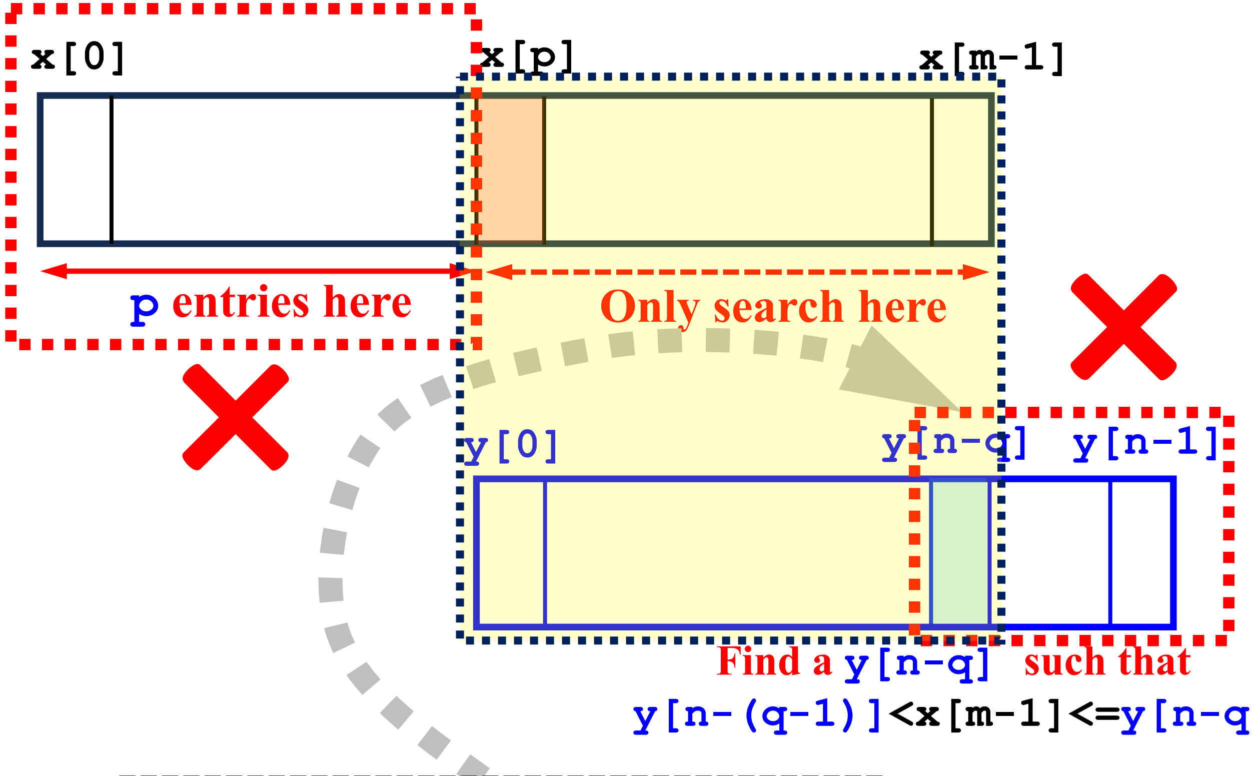
y[n-(q-1)]<x[m-1]<=y[n-q]

```
for (j = n-1; j >= 0; j--)
   if (y[j] < x[m-1]) {
      q = n-j-1;
      break;
   }
```

1. Because there are *m-p* entries in **x[]**, the total number of comparisons is (*m-p*)×(*n-q*).
2. We also need *p* comparisons to cut **x[]** and *q* comparisons for the **y[]**.
3. The total number of comparisons is (*m-p*)×(*n-q*) +(*p*+*q*).
4. The worst case is still *O*(*m*×*n*).
5. There is **no improvement in the worst case!**

Improvements? 5/5

Find a x[p] such that x[p-1] < y[0] <= x[p]



Find a y[n-q] such that y[n-(q-1)]<x[m-1]<=y[n-q]

```
for (j = n-1; j >= 0; j--)
   if (y[j] < x[m-1]) {
      q = n-j-1;
      break;
   }
```

1. These modifications indeed can speed up the original version.
2. However, in doing so **does not** improve the worst-case in a big way.
3. Therefore, we have to think differently to break the $O(m\times n)$ barrier.

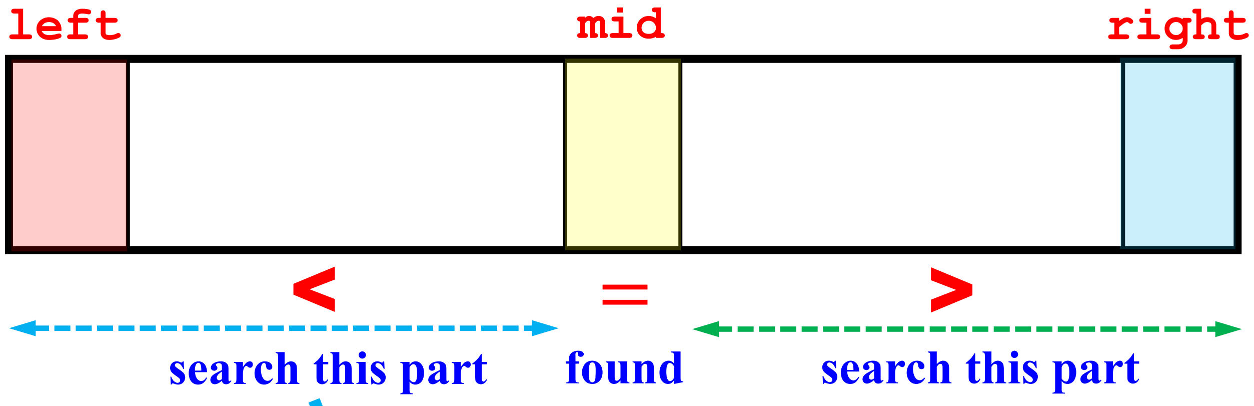
Is there a better way?

Of course, the immediate answer could be the **BINARY SEARCH**, if you have reached the data structures course.

Basic Idea: 1/2

1. We have an array `y[n]` whose elements are **distinct** and **sorted in ascending** order.
2. Is a given data item **DATA** in the array? If it is, **where is it**?

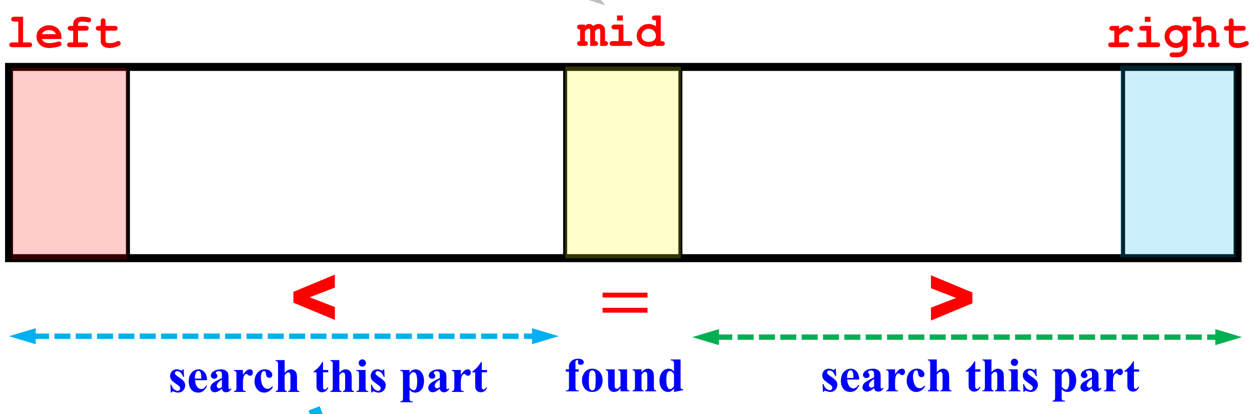
```
left  = 0;
right = n-1;
while (left <= right) {
   mid = (left + right)/2;
   if (DATA == y[mid]) {
      // found at mid
      // return mid
   }
   else if (DATA < y[mid])
      right = mid - 1;
   else
      left  = mid + 1;
   }
}
// return not-found
```



Basic Idea: 2/2

```
left  = 0;
right = n-1;
while (left <= right) {
   mid = (left + right)/2;
   if (DATA == y[mid]) {
      // found at mid
      // return mid
   }
   else if (DATA < y[mid])
      right = mid - 1;
   else
      left  = mid + 1;
   }
}
// return not-found
```

1. Because half of the elements is cut after each comparison, in no more than $\log_2(n)$ ***iterations*** we find the item or nothing.
2. The worst complexity is $O(\log_2(n))$



Solution: 1/6

```
int EQUAL_PAIRS(int x[], int y[],
        int m, int n, int xx[], int yy[])
{
    int i, j, k = 0;
    int left, right, mid;

    for (i = 0; i < m; i++) {
        left  = 0;   right = n-1;
        while (left <= right) {
            mid = (left + right)/2;
            if (x[i] == y[mid]) {
                xx[k] = i;  yy[k] = mid;
                k++;
                break;
            }
            else if (x[i] < y[mid])
                right = mid - 1;
            else
                left  = mid + 1;
        }
    }
    return k;
}
```

binary search

1. **This a possible solution.**
2. **For each `x[i]`, search array `y[]` to find it.**
3. **If `x[i]` is found, save positions to `xx[]` and `yy[]` and increase the count of equal pairs (`int k`).**
4. **Each iteration requires at most 2 comparisons.**
4. **Because each search requires $O(\log_2(n))$ comparisons, the total number of comparisons is $O(m\times\log_2(n))$.**

Solution: 2/6

```
int EQUAL_PAIRS(int x[], int y[],
      int m, int n, int xx[], int yy[])
{
    int i, j, k = 0;
    int left, right, mid;

    for (i = 0; i < m; i++) {
       left  = 0;    right = n-1;
       while (left <= right) {
          mid = (left + right)/2;
          if (x[i] == y[mid]) {
              xx[k] = i;  yy[k] = mid;
              k++;
              break;
          }
          else if (x[i] < y[mid])
              right = mid - 1;
          else
              left  = mid + 1;
       }
    }
    return k;
}
```

binary search

1. Use **x[i]** to search **y[]**,
   complexity: $O(m \times \log_2(n))$.
2. Use **y[j]** to search **x[]**,
   complexity: $O(n \times \log_2(m))$.
3. **Which way is better?**
4. Of course, one should choose the longer array (i.e., the larger of **m** and **n**) to be searched by the shorter array.
6. **Any justification?**

Solution: 3/6

x	$\log_2(x)$	Rounded Up
1	0	0
10	3.322	4
100	6.644	7
1,000	9.966	10
10,000	13.288	14
100,000	16.610	17
1,000,000	19.932	20
10,000,000	23.253	24

1. The left table shows the values of x and $\log_2(x)$.
2. It is clear that the increase of $\log_2(x)$ is much slower than that of the x.
3. Therefore, using the shorter array to search the longer array appears to be more efficient.
4. **We need a proof rather than an observation!**

Solution: 4/6

$$m\log_2(n) \overset{?}{<=>} n\log_2(m)$$

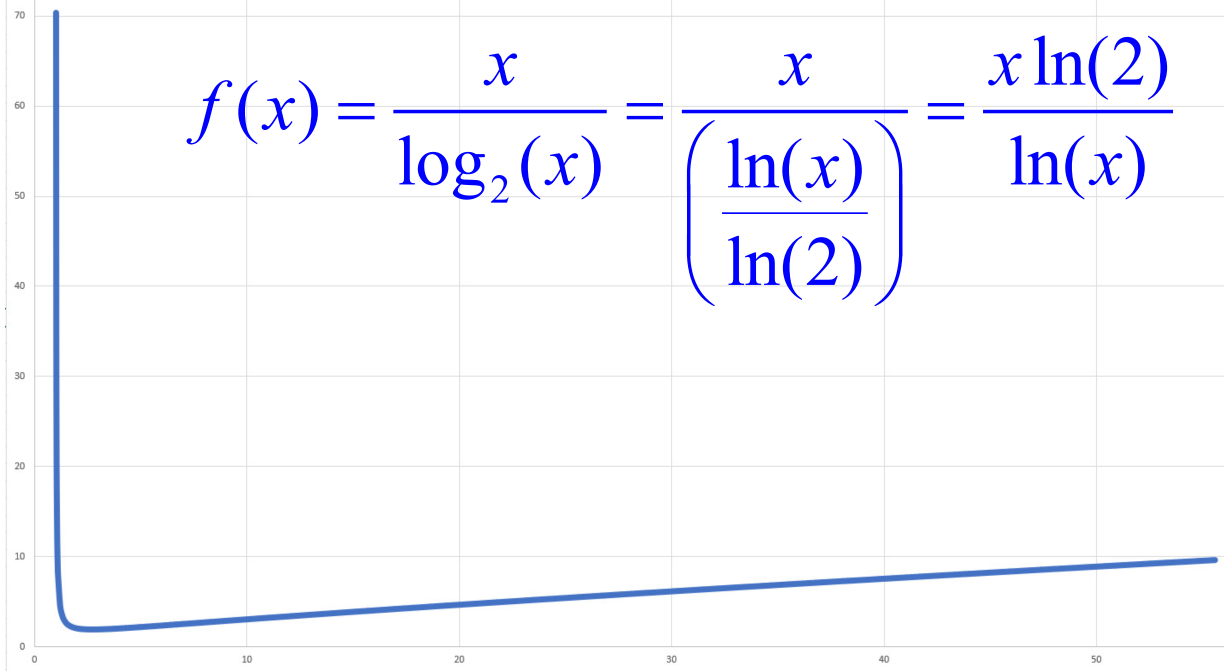
$$\frac{m}{\log_2(m)} <=> \frac{n}{\log_2(n)}$$

Let $f(x)=\dfrac{x}{\log_2(x)}$

If function $f(x)$ is an increasing one, then $m < n$ means $m/\log_2(m) < n/\log_2(n)$ and in turn it gives $m\log_2(n) < n\log_2(m)$.

1. **We want to know whether using the shorter array to search the longer one ($m\log_2(n)$) uses less comparisons than using the longer array to search the shorter one ($n\log_2(m)$).**
2. **Divide both sides by $\log_2(n)\times\log_2(m)$.**
3. **We have a function $f(x)=x/\log_2(x)$.**

Solution: 5/6



$$f(x)=\frac{x}{\log_2(x)}=\frac{x}{\left(\frac{\ln(x)}{\ln(2)}\right)}=\frac{x\ln(2)}{\ln(x)}$$

1. **If x=1, $\log_2(1)$=0 and $f(x)$ is ∞.**
2. **Compute the derivative of $f(x)$:**

$$\frac{df}{dx}=\ln(2)\frac{d}{dx}\left(\frac{x}{\ln(x)}\right)$$

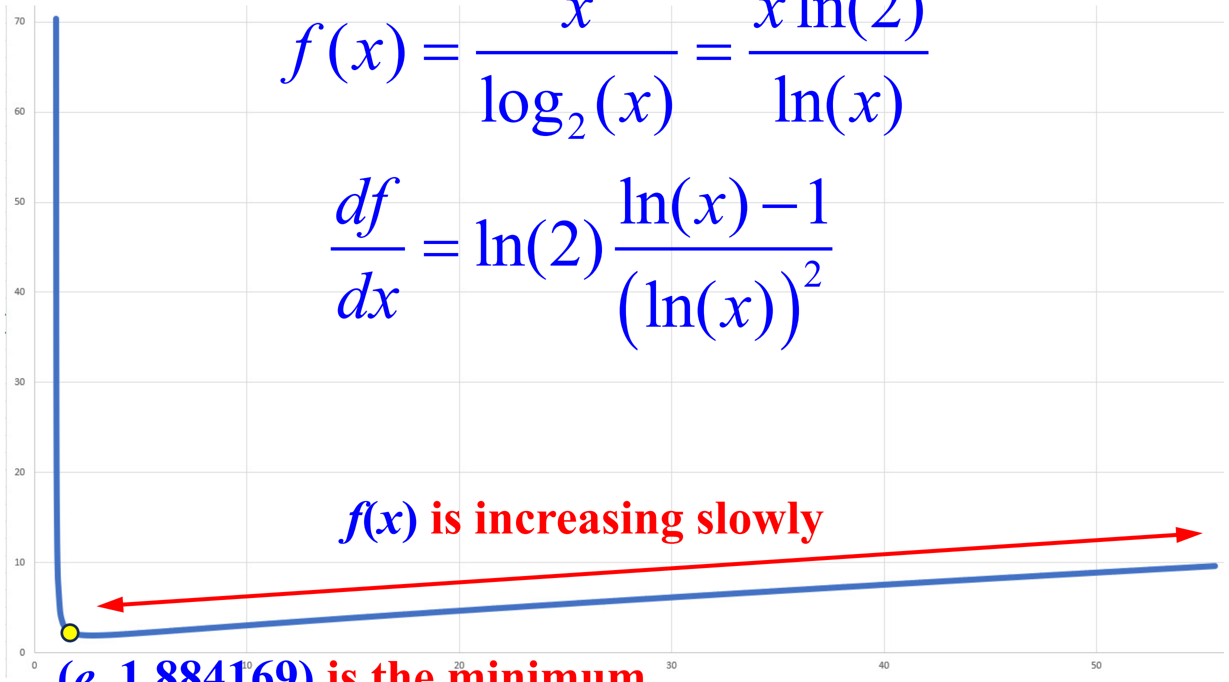
$$=\ln(2)\frac{\ln(x)\frac{dx}{dx}-x\frac{d(\ln(x))}{dx}}{\left(\ln(x)\right)^2}$$

$$=\ln(2)\frac{\ln(x)-1}{\left(\ln(x)\right)^2}$$

$$\frac{d(\ln(x))}{dx}=\frac{1}{x}$$

If function $f(x)$ is an increasing one, then $m < n$ means $m/\log_2(m) < n/\log_2(n)$ and in turn it gives $m\log_2(n) < n\log_2(m)$.

Solution: 6/6



$$f(x) = \frac{x}{\log_2(x)} = \frac{x\ln(2)}{\ln(x)}$$

$$\frac{df}{dx} = \ln(2)\frac{\ln(x)-1}{\left(\ln(x)\right)^2}$$

f(*x*) is increasing slowly

(*e*, 1.884169) is the minimum

1. The minimum of $f(x)$ is at $x=e$ because $f'(e)=0$!
2. $f(e) = e/\log_2(e)=1.884169$.
3. (e, 1.884169) is the minimum.
4. If $x > e$, $f(x)$ is increasing!
5. Hence, as long as m and n are larger than or equal to 2, using the shorter array to search the longer one is the way to go!

If function $f(x)$ is an increasing one, then $m < n$ means $m/\log_2(m) < n/\log_2(n)$ and in turn it gives $m\log_2(n) < n\log_2(m)$.

Improvements: 1/3

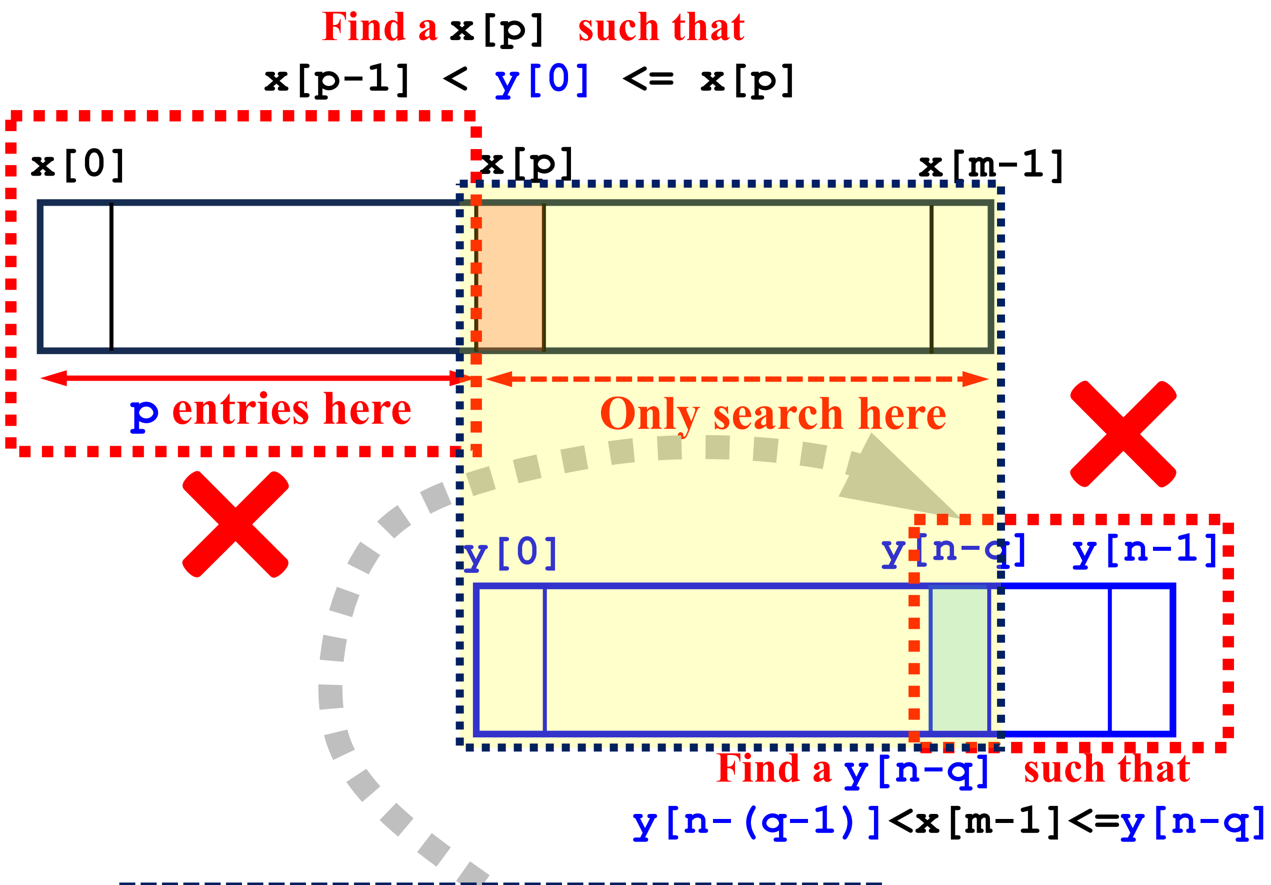
```
int EQUAL_PAIRS(int x[], int y[],
     int m, int n, int xx[], int yy[])
{
   int i, j, k = 0;
   int left, right, mid;

   for (i = 0; i < m; i++) {
      if (y[0] <= x[i] &&
          x[i] <= y[n-1]) {
         //
         // do a binary search
         //
   }
```

binary search

1. An obvious improvement is that if `x[i]` is not in the range of `y[0]` and `y[n-1]` , then **DO NOT SEARCH**.
2. This is because if `x[i]` is not in the range of `y[]`, you won't find it in `y[0..n-1]`.
3. In doing so, the worst case is still the same. For example:
   `x[]` = { 1, 3, 5, 7, 9}
   `y[]` = { 0, 2, 4, 6, 8, 10, 12}.

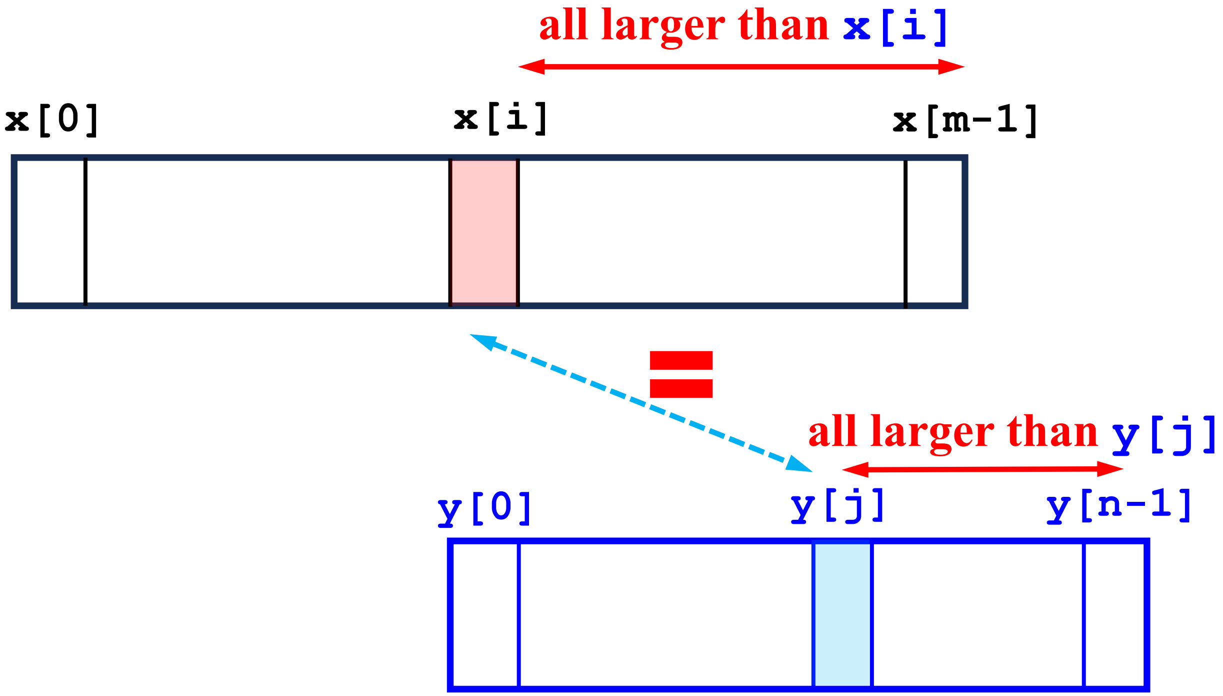
Improvements: 2/3

1. Just like in the previous solution, we could ignore portions of arrays `x[]` and/or `y[]` to cut down the number of comparisons.
2. However, this **does not** improve the worst case because *p* and *q* could be 0!



```
for (j = n-1; j >= 0; j--)
   if (y[j] < x[m-1]) {
      q = n-j-1;
      break;
   }
```

Improvements: 3/3

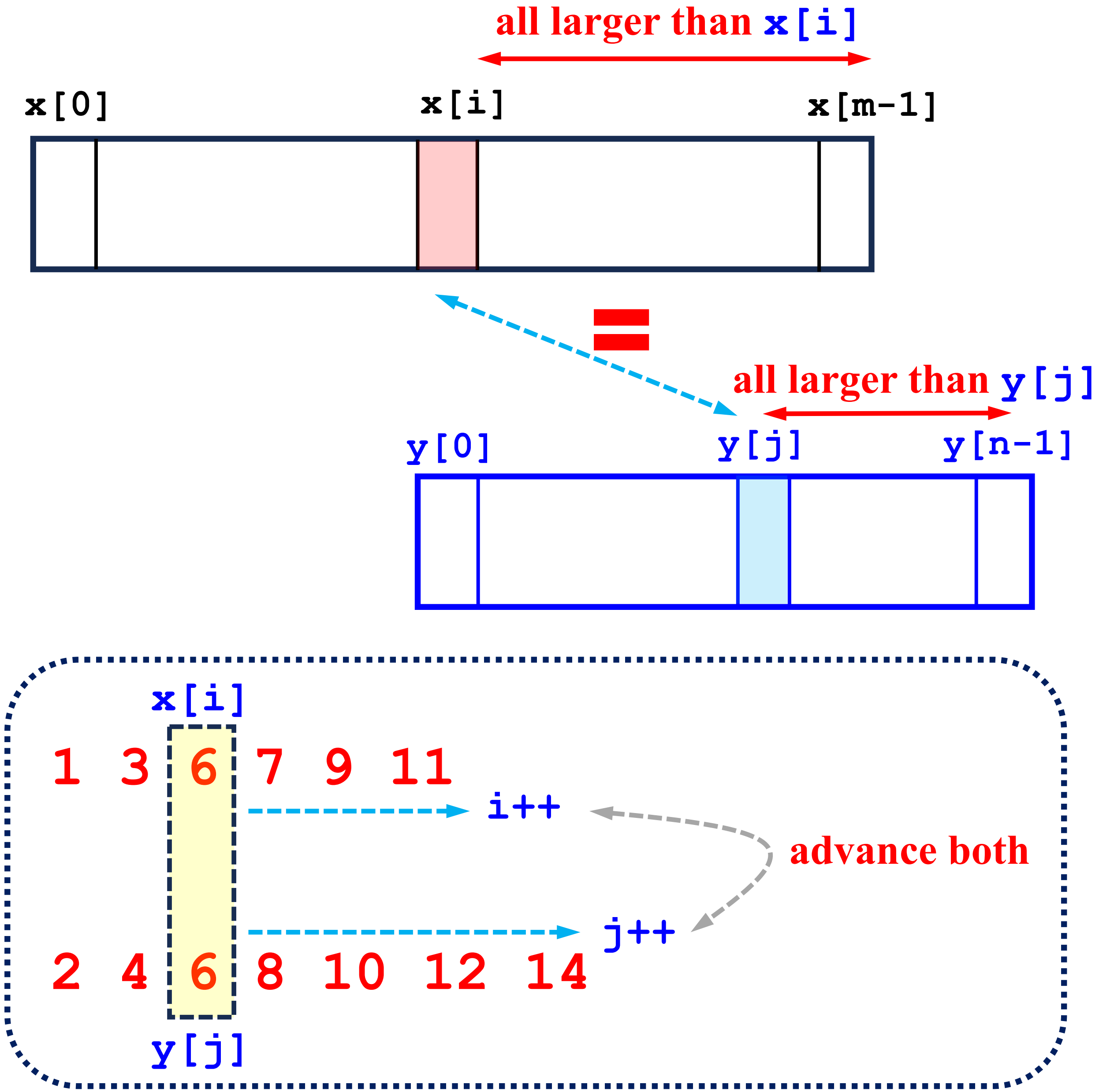


1. If `x[i]` is equal to `y[j]`, then elements in `x[i+1..m-1]` can not be found in `y[0..j]`.
2. This is because `x[]` and `y[]` are **sorted** with **distinct** data.
3. Hence, if `x[i] = y[j]`, the search range of `x[i+1]` is `y[j+1]` to `y[n-1]`.
4. It does not improve the worst case complexity.
5. It is an important observation for developing an optimal solution.

An Optimal Solution

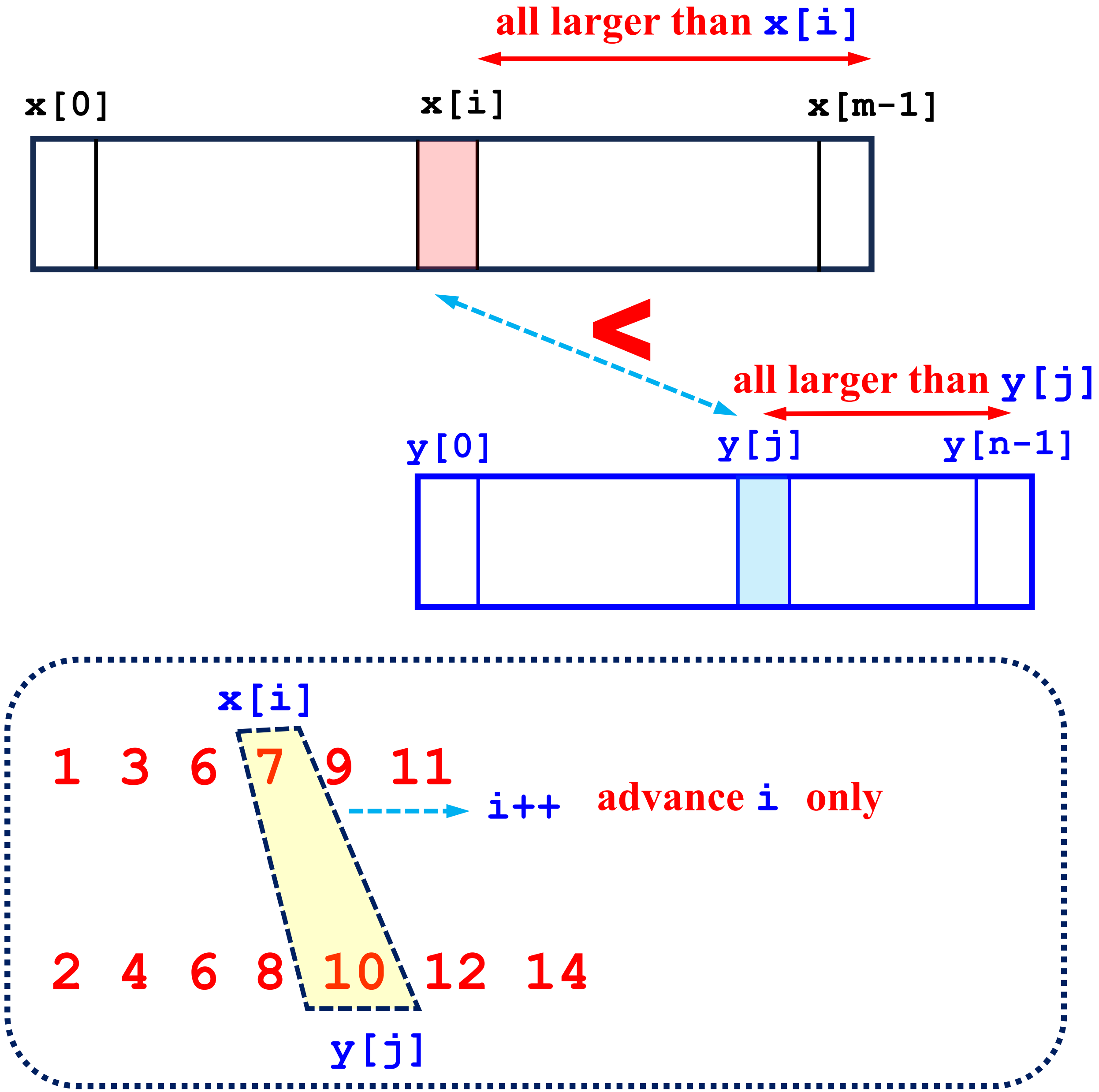
We have all the needed tools to do this

Idea: 1/5



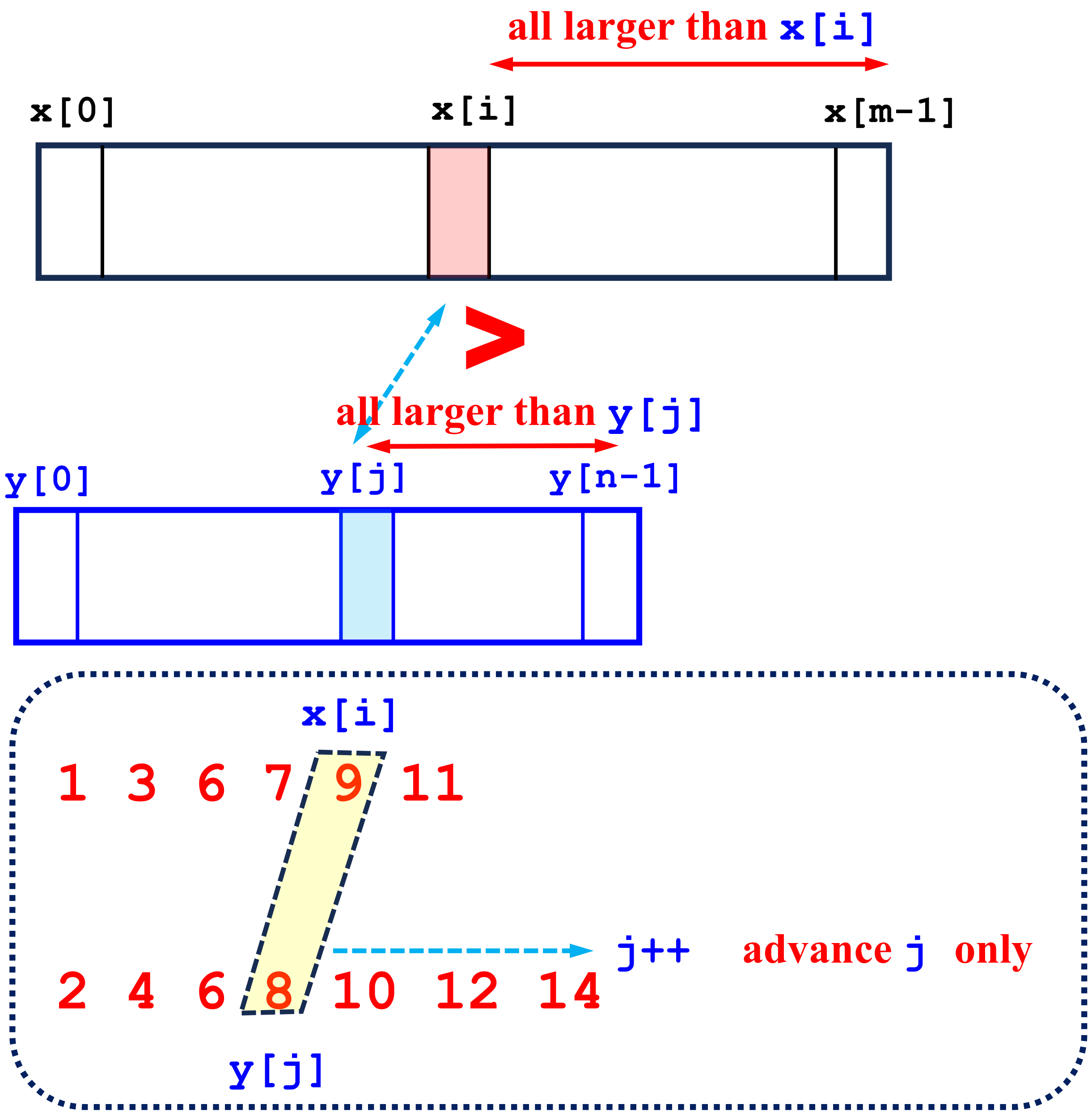
1. If `x[i]` = `y[j]`, then elements in `x[i+1..m-1]` cannot be found in `y[0..j]`.
2. Similarly, `y[j+1..n-1]` cannot be found in `x[0..i]`.
3. This is because `x[]` and `y[]` are **sorted** with **distinct** data.
4. Hence, if `x[i] = y[j]`, the search range would be restricted to `x[i+1..m-1]` and `y[j+1..n-1]`.

Idea: 2/5



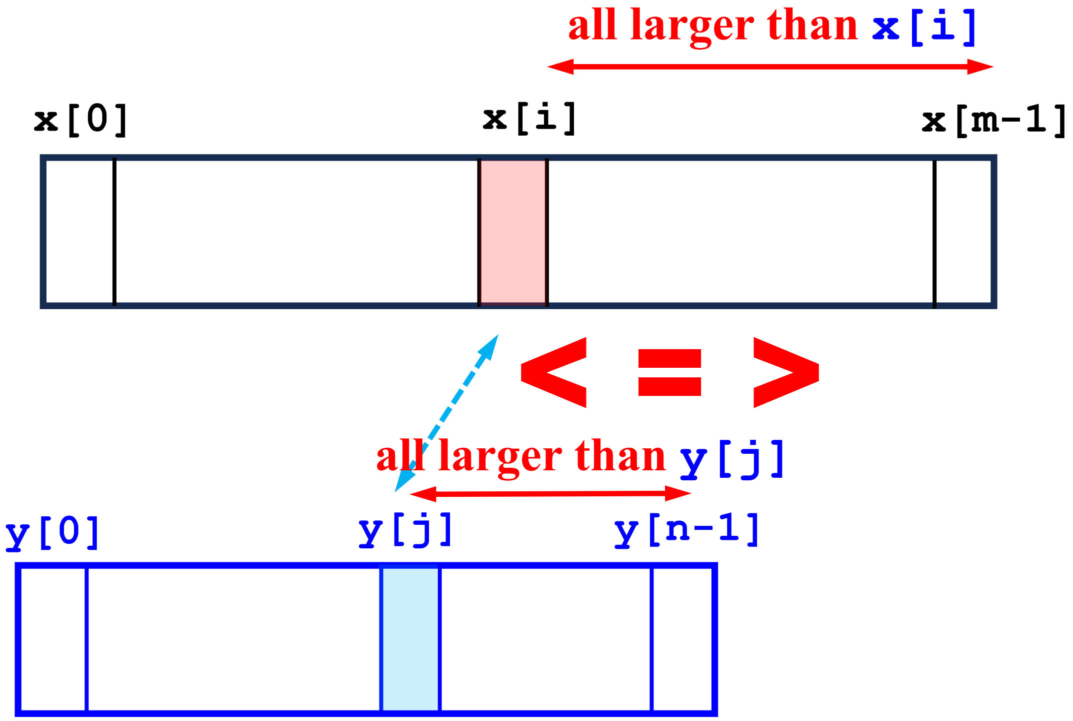
1. If `x[i] < y[j]`, what is the next step?
2. `x[i]` cannot be found in `y[j..n-1]`.
3. Therefore, try the next element `x[i+1]`.
4. Consequently, if `x[i] < y[j]`, what we need to do is `i++`.

Idea: 3/5



1. If `x[i] > y[j]`, what is the next step?
2. `y[j]` cannot be found in `x[i..m-1]`.
3. Therefore, try the next element `y[j+1]`.
4. Consequently, if `x[i] > y[j]`, what we need to do is `j++`.

Idea: 4/5



1. We have three cases:
   a) If `x[i]` < `y[j]`, set `i++` to advance `x[i]` to the next.
   b) If `x[i]` > `y[j]`, set `j++` to advance `y[j]` to the next.
   a) If `x[i]` = `y[j]`, set `i++` and `j++` to advance both `x[i]` and `y[j]` to the next.
2. In this way, two comparisons per iteration are needed!

Idea: 5/5

```
i = j = 0;
while (i < m && j < n) {
    //
    // do the comparisons
    //
}
```

1. `i` and `j` must start with 0!
2. As long as there are `x[i]` and `y[j]` in the arrays, the process continues.
3. Therefore, the loop structure looks like this:

```
int EQUAL_PAIRS(int x[], int y[],
         int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               xx[k] = i;
               yy[k] = j;
               i++;
               j++;
               k++;
          }
     }
     return k;
}
```

Solution

1. **We have three cases:**
   a) **If `x[i]` < `y[j]`, set `i++` to advance `x[i]` to the next.**
   b) **If `x[i]` > `y[j]`, set `j++` to advance `y[j]` to the next.**
   c) **If `x[i]` = `y[j]`, set `i++` and `j++` to advance both `x[i]` and `y[j]` to the next.**

```
int EQUAL_PAIRS(int x[], int y[],
                int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               i++;
               j++;
          }
     }
     return k;
}
```

Analysis: 1/5

1. **How many comparisons?**
2. If `x[i] < y[j]`, then `i++` and uses 1 comparison.
3. If `x[i] > y[j]`, then `j++` and uses 2 comparisons.
4. If `x[i] = y[j]`, then `i++` and `j++` and uses 2 comparisons.

```
int EQUAL_PAIRS(int x[], int y[],
         int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               i++;
               j++;
          }
     }
     return k;
}
```

Analysis: 2/5

1. **How many comparisons?**
2. If `i` takes *m* moves and `j` takes *n* moves, the total number of comparisons is *m*+2*n*!
3. **Is this possible?**
4. **Yes**, it is possible.
5. If `x[m-1]` is equal to `y[n-1]`, then "`i` takes *m* moves and `j` takes *n* moves."
6. This requires *m*+2*n* comparisons.
7. This is a *O*(*m*+2*n*) solution.

Analysis: 3/5

```
int EQUAL_PAIRS(int x[], int y[],
                int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               i++;
               j++;
          }
     }
     return k;
}
```

1. Assume $m < n$.
2. If `x[i] = y[i]` for all `i`, then each `j++` costs **2** comparisons and the total number of comparisons is exactly $2m$.

Analysis: 4/5

```
int EQUAL_PAIRS(int x[], int y[],
         int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               i++;
               j++;
          }
     }
     return k;
}
```

1. Assume *m* < *n*.
2. If `x[m-1] < y[0]`, then only *m* comparisons are needed.
3. On the other hand, if `x[0] > y[n-1]`, then every comparison is a "`>`", the total number of comparisons is 2*n*.

```
int EQUAL_PAIRS(int x[], int y[],
         int m, int n, int xx[], int yy[])
{
     int i, j, k;

     i = j = k = 0;
     while (i < m && j < n) {
          if (x[i] < y[j])
               i++;
          else if (x[i] > y[j])
               j++;
          else {
               i++;
               j++;
          }
     }
     return k;
}
```

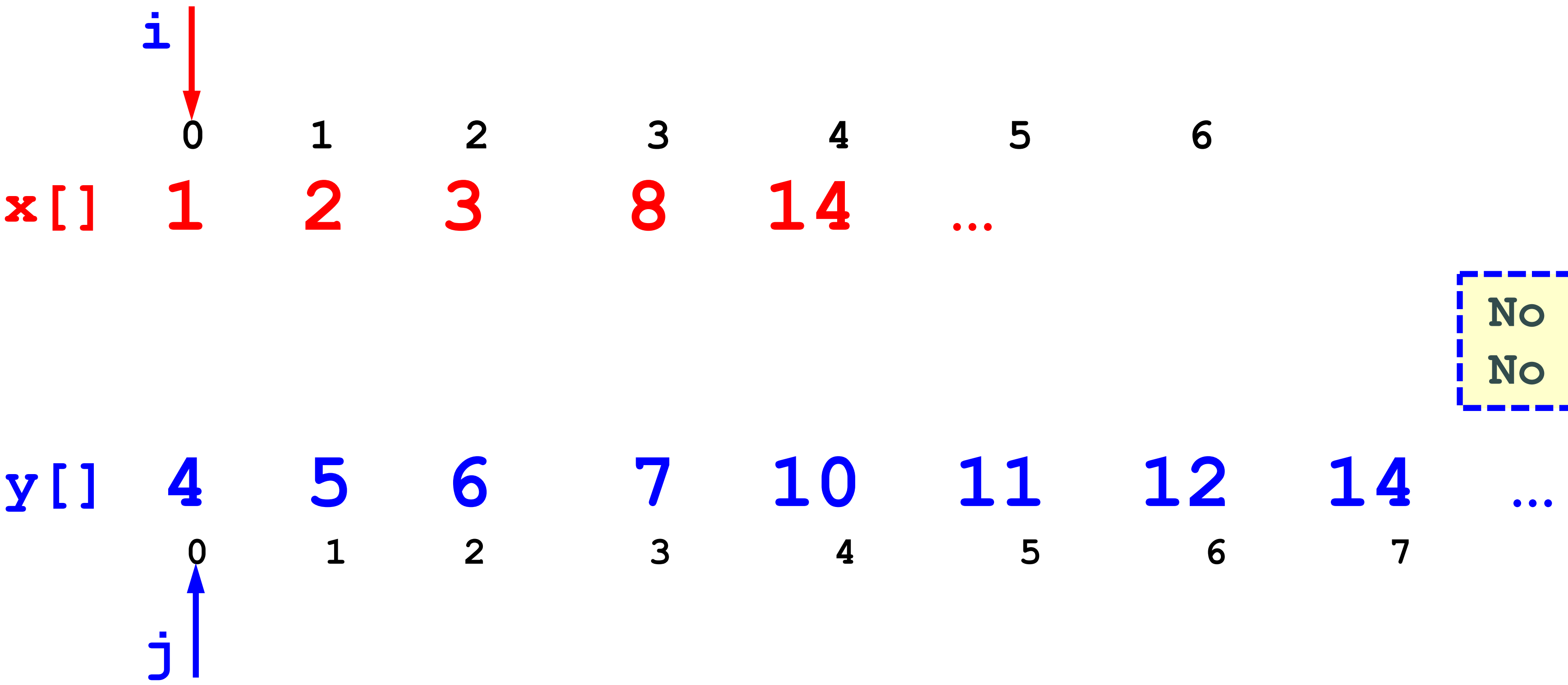
Analysis: 5/5

1. Assume *m* < *n*.
2. Therefore, call `EQUAL_PAIRS()` with the longer array as the `x[]` and the shorter array as `y[]` so that *m*+2*n* is smaller.

Example: 1/5

```
< Comparison steps   : 0
> Comparison steps   : 0
♦ Previous total      : 0
♦ Total comparisons  : 0
```

- Initially, `i` and `j` are both 0.



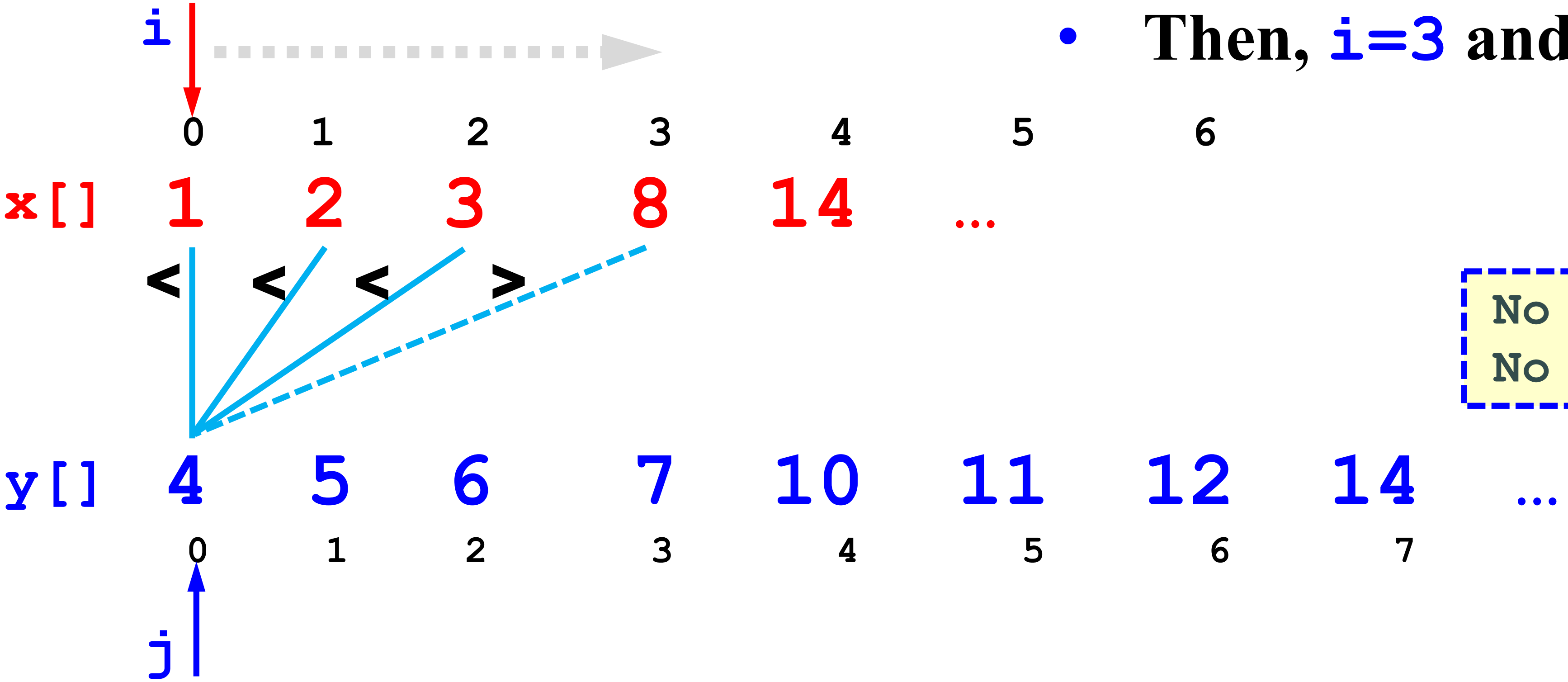
```
        i
        0    1    2    3    4    5    6
x[]     1    2    3    8    14   …

y[]     4    5    6    7    10   11   12   14   …
        0    1    2    3    4    5    6    7
        j
```

```
No of <       : 0
No of > & =   : 0
```

Example: 2/5

```
< Comparison steps   : 3
> Comparison steps   : 1
♦ Previous total     : 0
♦ Total comparisons  : 3+1×2 = 5
```

- Because x[0]<y[0], do i++ until x[3]>y[0].
- We have 3 <s and 1 >.
- Then, i=3 and j moves to 1.

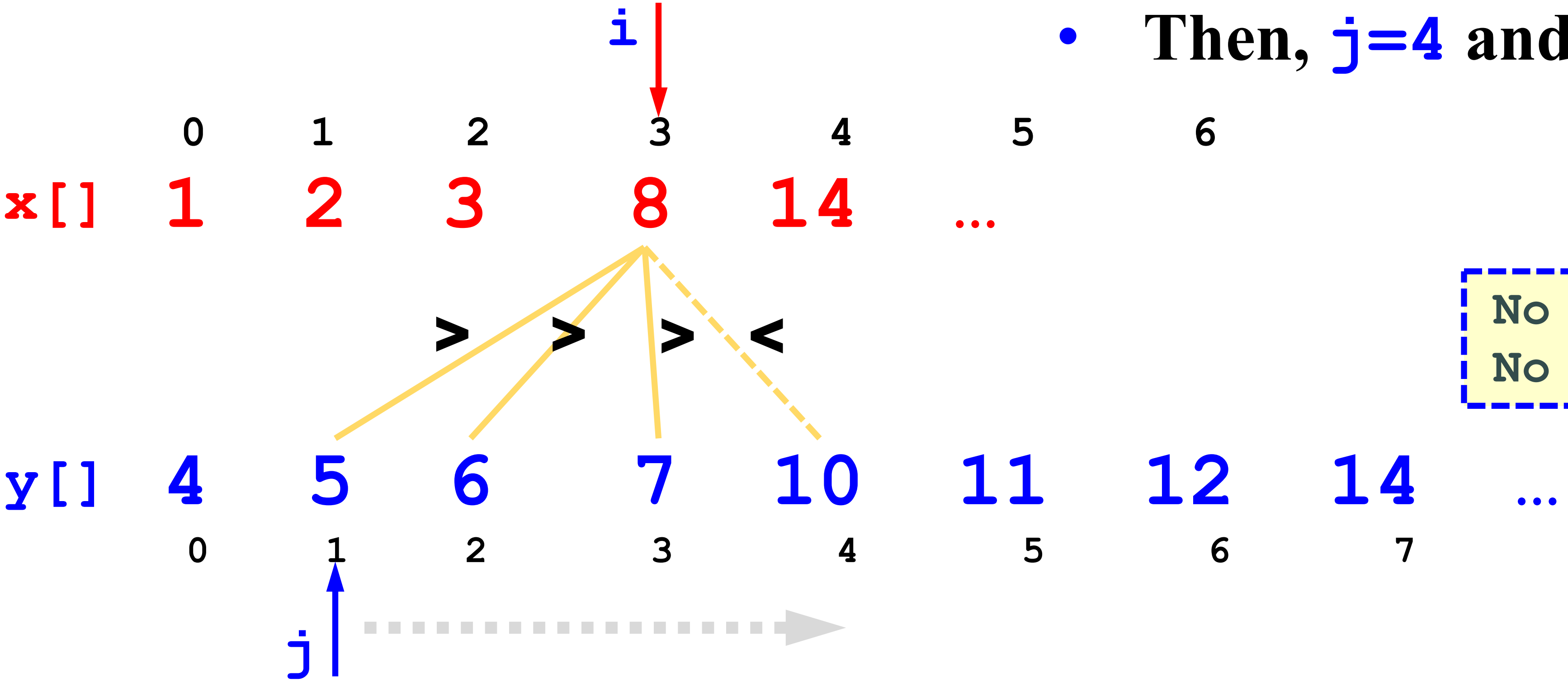


```
No of <       : 3
No of > & =   : 1
```

Example: 3/5

```
< Comparison steps   : 1
> Comparison steps   : 3
♦ Previous total     : 5
♦ Total comparisons  : 5+(1+3×2) = 12
```

- Because **x[3]>y[1]**, do **j++** until **x[3]<y[4]**.
- We have 3 >s and 1 <.
- Then, **j=4** and **i** moves to **4**.



i

```
       0   1   2   3   4   5   6
x[]    1   2   3   8   14  …

               >   >   >   <

y[]    4   5   6   7   10  11  12  14  …
       0   1   2   3   4   5   6   7
```

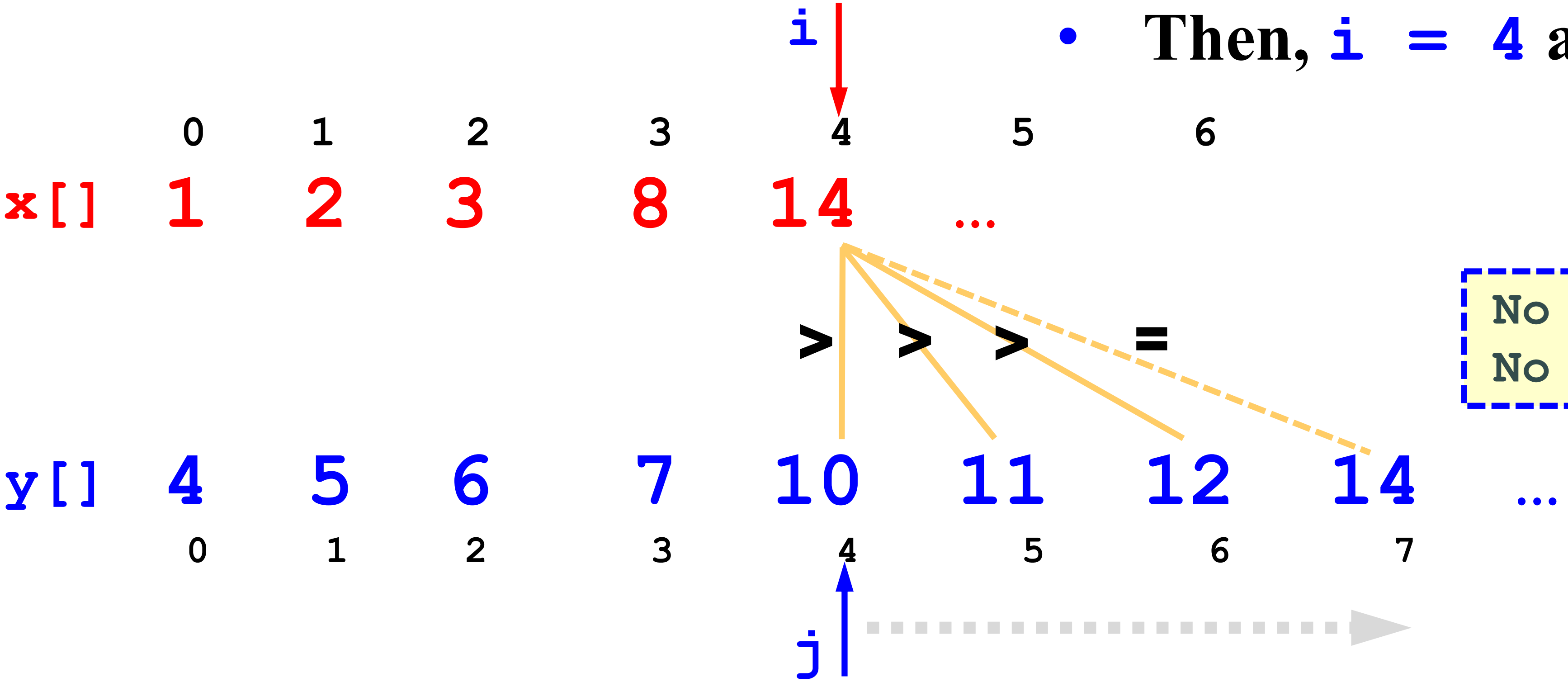
j

```
No of <       : 1
No of > & =   : 3
```

Example: 4/5

```
< Comparison steps   : 1
> Comparison steps   : 1
♦ Previous total     : 12
♦ Total comparisons  : 12+(0+4×2) = 20
```

- Because x[4]>y[4], do j++ until x[4] = y[7].
- We have 3 >s and 1 =.
- Then, i = 4 and j = 7.

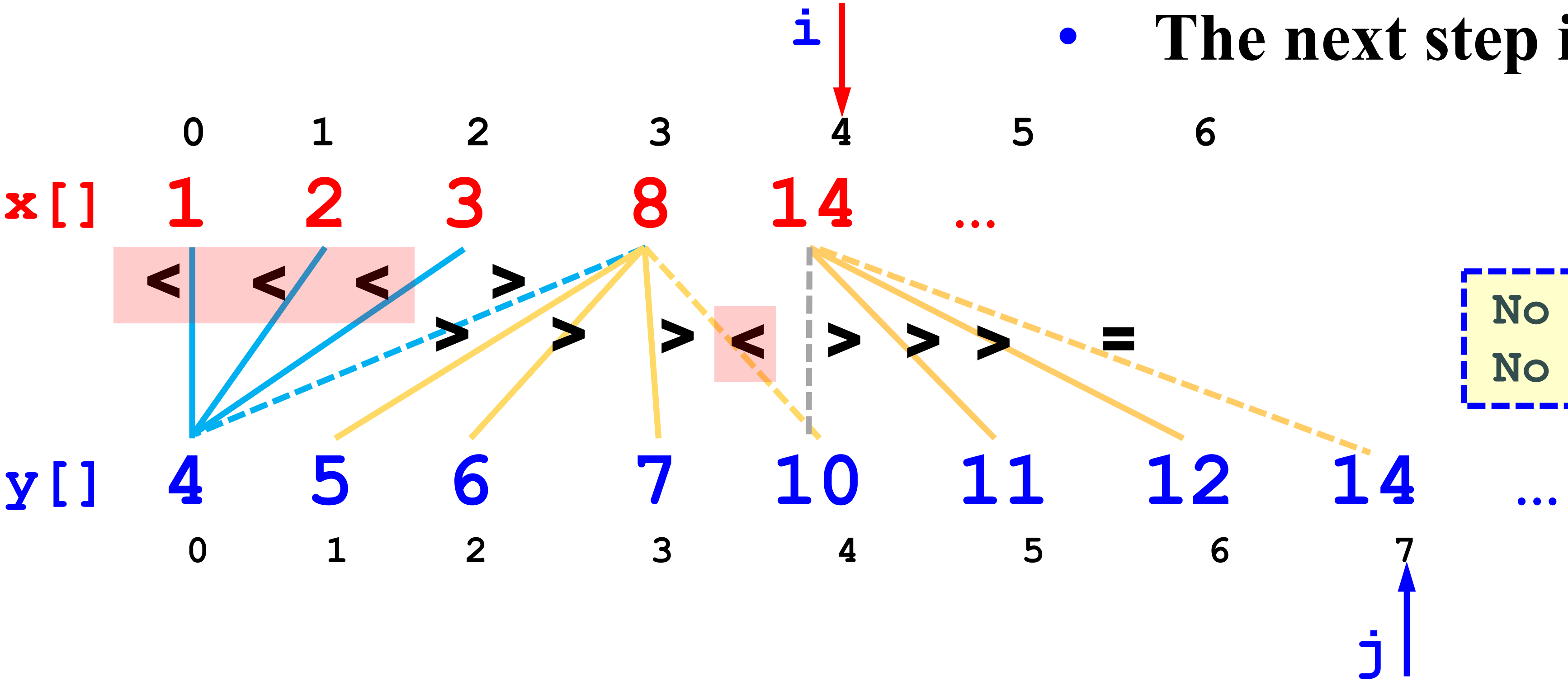


```
No of <       : 0
No of > & =   : 4
```

Example: 5/5

♦ Total comparisons : 20

- The total number of comparisons is 20 = 4(<)+2×8(>).
- This is equal to $m+2n$ = 4+2×8.
- The next step is `i++` and `j++`.



```
No of <       : 4
No of > & =   : 8
```

Improvements?

- □ It is easy to avoid unnecessary comparisons when `x[0] > y[n-1]` or `x[m-1] < y[0]`.
- □ The technique to remove some unnecessary comparisons discussed on Slides 11--15 may also be used.
- □ These could be minimal with respect to complexity.
- □ If p and q comparisons are used to trim the first and second parts of `x[]` and `y[]`, respectively, the number of comparisons is $(p+q)+[(m-p)+2(n-q)] = (m+2n)-q$.

A Summary

What did we learn?

- This **EQUAL PAIR** problem is a good programming problem for beginners.
- It could be a little bit challenging if **not all given conditions are used** properly and fully.
- We started with a naïve $O(m \times n)$ solution, moved on to a better one $O(\min(m,n)\log_2(\max(m,n))$ and finally reached an optimal one $O(m+2n)$.

Food for Thought

- ❑ What if the **distinct** condition is dropped? For example, if `x[3..5]` contains 4, 4 and 4 and `y[7..8]` has 4 and 4, then one should report `x[3]` and `y[7]` and `x[3]` and `y[8]`, `x[4]` and `y[7]` and `x[4]` and `y[8]`, and `x[5]` and `y[7]` and `x[5]` and `y[8]`. Modify the solution to do the same?
- ❑ What if the **sorted** condition is dropped? Well, one could sort both arrays and the solution cannot be linear! **Why?**

References

1. M. Rem, Small Programming Exercise 2, *Science of Computer Programming*, Vol. 3 (1983), pp. 313—319.

The End

Happy Programming!