Chapter 10 Section 1

MA1020 Quantitative Literacy

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Basics

- Experiment
- Outcome
- Sample Space
- Event

Properties

For sample space S and events A and B,

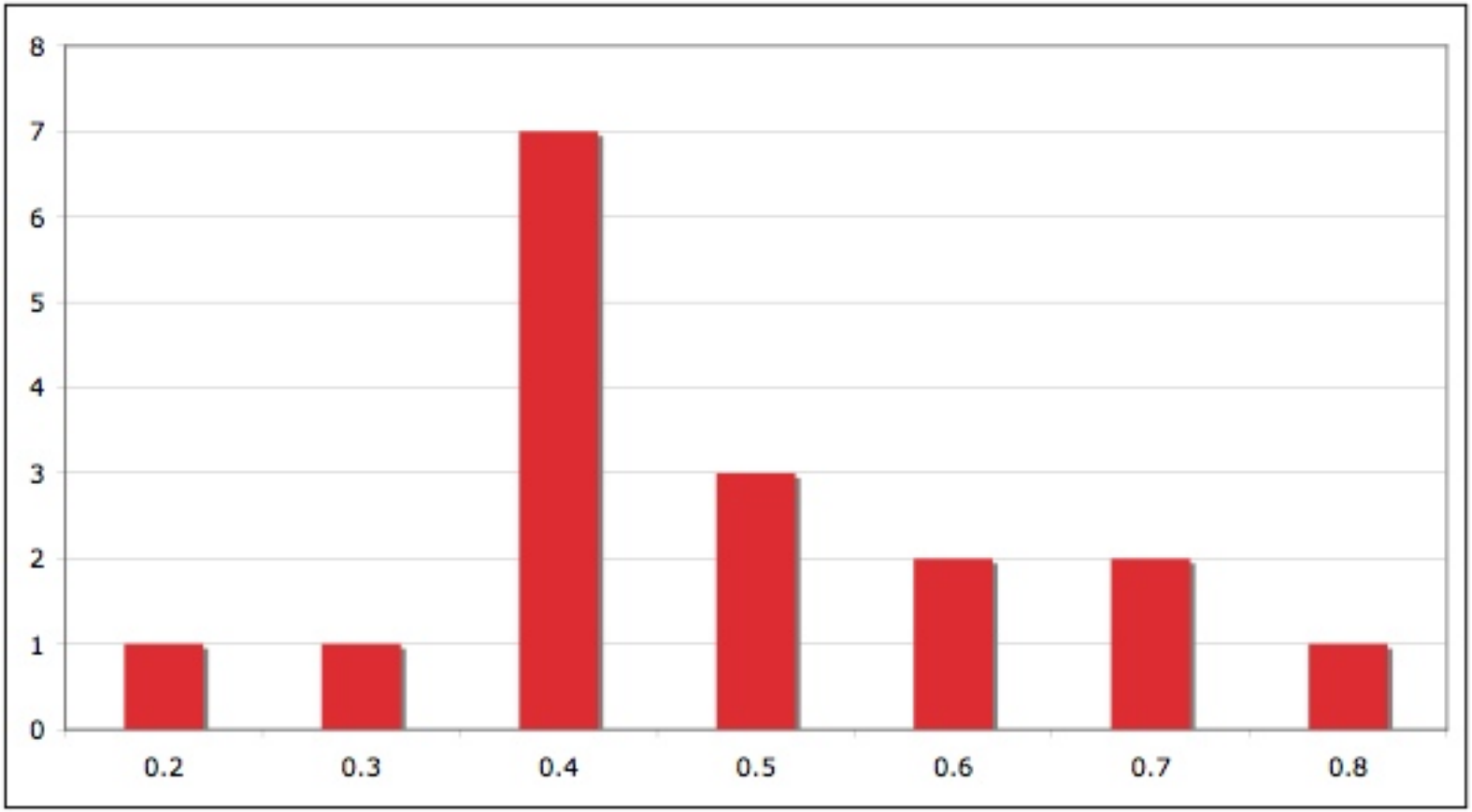
1. For any event A, $0 \leq P(A) \leq 1$.
2. $P(\emptyset) = 0$.
3. $P(S) = 1$.

Exercise #12

An experiment consists of flipping a coin and rolling an eight-sided die and noting whether the coin lands with a head or tail showing and which number faces up on the die. The eight faces on the die are labeled 1, 2, 3, 4, 5, 6, 7, and 8. List each of the following:

1. The sample space
2. The event that a 2 faces up on the die
3. The event that the coin lands with a head showing
4. The event that a coin lands with a tail showing and an odd number faces up on the die
5. The event that the coin lands with a head showing or a 7 faces up on the die.

Penny Experiment



Theoretical Probability

Definition

Suppose that all the outcomes in the sample space S are equally likely to occur. Let E be an event. Then the probability of event E, denoted $P(E)$, is

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in } S}.$$

Exercise #22

An experiment consists of rolling an eight-sided die and a standard six-sided die and noting the numbers that show on the top faces. Assume the dice are fair.

1. List the elements in the sample space.
2. Find the theoretical probability of the event that the sum of the two numbers is greater than 6.
3. Find the theoretical probability of the event that the sum of the two numbers is less than 7.
4. Find the theoretical probability of the event that the product of the two numbers is a multiple of 5.

Mutually Exclusive Events

Definition

The union of two events $A \cup B$ refers to all outcomes that are in one, or the other, or both events.

Definition

The intersection of two events $A \cap B$ refers to outcomes that are in both events.

Definition

Events which have no outcome in common are mutually exclusive.

Probability of Mutually Exclusive Events

If L and M are mutually exclusive events, then $P(L \cup M) = P(L) + P(M)$.

Exercise #32

For an experiment in which a fair coin is tossed and a fair standard die is rolled, consider the following events.

H : The coin lands heads up.

F : The die shows a number greater than 4.

1. Find $P(H)$ and $P(F)$.
2. Find and interpret $P(H \cup F)$ and $P(H \cap F)$.
3. Are events H and F mutually exclusive.

Complement of an Event

Probability of an Event and its Complement

The relationship between the probability of an event E and the probability of its complement $\overline{E}$ is given by

$$P(E) = 1 - P(\overline{E}) \text{ and } P(\overline{E}) = 1 - P(E).$$

Exercise #38

Consider the experiment of randomly placing one silver dollar (D) and three rocks (R) inside four drawers so that one object is in each drawer. The results are recorded in order.

1. List all possible outcomes in the sample space.
2. Let E be the event the silver dollar is hidden in the first drawer. List the outcome(s) of the sample space that corresponds to event E.
3. Describe $\overline{E}$ and list the outcome(s) of the sample space that corresponds to $\overline{E}$.
4. Find $P(E)$ and $P(\overline{E})$.

Union of Two Events

Probability of the Union of Two Events

If A and B are any two events from a sample space, S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Exercise #34

Suppose a jar contains 20 marbles, numbered 1 through 20, with each odd-numbered marble colored red, and each even-numbered marble colored lack. A marble is drawn from the jar and its color and number are noted.

1. List the sample space.
2. Consider the following events.

$$A: \text{ getting a black marble}$$

$$B: \text{ getting a number divisible by 3}$$

   Find $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A \cup B)$.
3. Verify that the equation $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for the probabilities in problem 2.