

Chapter 3 Section 2

MA1020 Quantitative Literacy

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Fairness Criteria

- Majority Criterion
- Head-to-Head Criterion
- Monotonicity Criterion
- Irrelevant-Alternatives Criterion

Majority Criterion

Definition

If a candidate is the first choice of a majority of voters, then that candidate should be selected.

Majority Criterion

Example

A national association of bicycle manufacturers is planning its annual trade show. The steering committee, which consists of nine members, is considering four cities for the event: Chicago (C), Phoenix (P), and Boston (B).

Table 3.16

Ranking	Number of Planning Committee Members				
	3	2	2	1	1
1st	C	S	C	P	S
2nd	S	B	P	S	B
3rd	B	P	S	C	C
4th	P	C	B	B	P

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Head-to-Head Criterion

Definition

If a candidate is favored when compared separately with each of the other candidates, then the favored candidate should be elected.

Head-to-Head Criterion

Example

A seven-member accounting department is planning a retirement celebration for a fellow employee. Three options have been discussed: a catered meal at the office (C), a picnic and barbecue (P), and a restaurant dinner (R).

Table 3.17

Ranking	Number of Accounting Department Members		
	3	2	2
1st	P	R	C
2nd	R	C	R
3rd	C	P	P

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Monotonicity Criterion

Example

Forty-one elementary teachers in a local school district must choose between three candidates for union president: Akst (A), Bailey (B), and Chung (C).

Table 3.18

Ranking	Number of Teachers			
	14	12	5	10
1st	A	B	C	C
2nd	C	A	A	B
3rd	B	C	B	A

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Monotonicity Criterion

Definition

Suppose a particular candidate X , is selected in an election. If hypothetically, this election were to be held again and each voter who changes his or her preferences does so by switching the positions of X and the candidate of one position above X in that voter's preference ranking, then the candidate X should still be selected.

Irrelevant-Alternatives Criterion

Example

A local book club with five active members is voting to decide which book to read next. Three book choices have been nominated: a mystery (M), a historical novel (H), and a science fiction fantasy (S).

Table 3.22

Ranking	Number of Book Club Members		
	2	1	2
1st	M	H	S
2nd	S	M	H
3rd	H	S	M

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Irrelevant-Alternatives Criterion

Definition

Suppose a particular alternative, X , is selected in an election. If, hypothetically, this election were to be held again, but with one or more of the unselected alternatives removed from consideration, then the alternative X should still be selected.

Arrow Impossibility Theorem

Table 3.25

Method	Majority Criterion Always Satisfied?	Head-to-head Criterion Always Satisfied?	Monotonicity Criterion Always Satisfied?	Irrelevant-Alternatives Criterion Always Satisfied?
Plurality	Yes	No	Yes	No
Borda count	No	No	Yes	No
Plurality with elimination	Yes	No	No	No
Pairwise comparison	Yes	Yes	Yes	No

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Approval Voting

Definition

Each voter votes for all the candidates he or she considers acceptable. The candidate with the greatest number of votes is selected.