Chapter 3 Section 3 MA1020 Quantitative Literacy

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Weighted Voting Systems

- Notation: P_n and W_n .
- Simple Majority
- Supermajority
- Quota

Definition

A coalition is a nonempty set of voters.

- Winning Coalition
- Losing Coalition

Suppose representatives for five zones have voting weights of 4, 6, 2, 8, and 10, respectively.

- If passing a motion requires a simple majority of yes votes, then what is the smallest weight required to pass a motion?
- If passing a motion requires a two-thirds supermajority of yes votes, then what is the smallest weight required to pass a motion?
- If the quota is 25, then give the notation for this weighted voting system.



The following set of numbers represents the weights assigned to voters in a weighted voting system. Following the weight is the percentage required for measures to pass. Determine the quota and express the weighted voting system using the proper notation.

[8, 5, 5, 3, 3, 2]; 60%



Example

Consider the weighted voting system [14|5, 4, 3, 2]. What must happen in order to pass a motion?

Example

For the weighted voting system [16|9,7,6,4,3,2], determine if the following coalitions of voters are winning or losing coalitions? { P_1, P_4, P_6 } { P_2, P_3, P_6 } { P_2, P_3, P_4 } { P_3, P_4, P_5, P_6 }

Number of Coalitions

$$2^{n} - 1$$

Example

In a weighted voting system with 15 voters, how many coalitions are possible?

"Special" Voters

- Dictators
- Dummies
- Veto Power
- Critical Voter



Identify voters who are dictators, dummies or have veto power.

- [8|5,4,3]
- [25|14, 13, 12, 8]
- [7|7,2,2,2]

Banzhaf Power Index

- Banzhaf Power
- Total Banzhaf Power
- Banzhaf Power Index



For [6|5,3,1], find the Banzhaf power index for each voter.