Chapter 4 Section 1

MA1020 Quantitative Literacy

Sidney Butler

Michigan Technological University

September 27, 2006

Vocabulary

- fair-division problem
- players
- division procedure
- fair-division scheme
- fair share
- proportional

Types of Fair-Division Problems

- Continuous Fair-Division Problems
- Discrete Fair-Division Problems
- Mixed Fair-Division Problems

Assumptions

- The value of a player's share is determined by his or her own preferences or values.
- A player's values in a fair-division problem cannot change based on the results of the division.
- No player has knowledge of any other player's values.

Divide-And-Choose Method For Two Players

1. Player X divides the item into two pieces that he or she considers of equal value.
2. Player Y picks whichever of the two pieces he or she considers to be of greater value. If player Y also considers the pieces to be of equal value, then he or she can pick either piece.
3. Player X gets the piece that player Y did not select.

Divide-And-Choose Method for Three Players

1. Player X divides the item into three pieces of equal value to him or her.
2. Players Y and Z decide independently which pieces are worth at least on-third of the cake's value.
3. The choosers annouces which pieces they consider to be acceptable.
4. This step depends upon which pieces are declared acceptable to the player.
   a) If at least one piece is unacceptable to either player Y or player Z, then the divider, player X, gets one of those unacceptable pieces. If both player Y and Z can choose *different* acceptable pieces from the two remaining pieces, they do so. otherwise, they must put the remaining two pieces back together and use the divide-and-choose method for two players to divide those reassembled pieces.
   b) If every piece is acceptable to player Y and player Z, then both Y and Z take pieces they consider acceptable. The divider gets whichever piece is left.

The Last-Diminisher Method of Fair Division

Suppose any number of players X, Y, . . . are dividing an item. To give one player a piece that the player considers a fair share and that no other player considers to be more than fair, the procedure is as follows.

1. Player X cuts a piece of the item that he or she considers to be a fair share.
2. Each player in turn judges the fairness of the piece.
   a) If a player considers the piece to be a fair share or less than a fair share, then it is the next player's turn to judge the fairness of the piece.
   b) If a player considers the piece to be larger than a fair share, then that player trims the piece to a smaller size that he or she feels is a fair share. The trimmed-off piece is reattached to the main body of the item, and it is the next player's turn to judge the fairness of the just-trimmed piece.
3. The last player who trimmed the item to a smaller size gets the piece. If no player trimmed the cake, then player X, who originally cut the piece, gets the piece.

Repeat After one player takes a piece of cake, begin the whole process again without that player and that piece. When only two players are left, they use the divide-and-choose method for two players to divide the remaining piece fairly.